Nonlinear Unemployment Effects of the Inflation Tax*

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Abstract

We argue that long-run inflation has nonlinear and state-dependent effects on unemployment, output, and welfare. Using panel data from the OECD, we document three correlations. First, there is a positive long-run relationship between anticipated inflation and unemployment. Second, there is also a positive correlation between anticipated inflation and unemployment volatility. Third, the long-run inflation-unemployment relationship is not only positive, but also stronger when unemployment is higher. We show that these correlations arise in a standard monetary search model with two shocks – productivity and monetary – and frictions in labor and goods markets. Inflation lowers the surplus from a worker-firm match, in turn making it sensitive to productivity shocks or to further increases in inflation. We calibrate the model to match the US postwar labor market and monetary data and show that it is consistent with observed cross-country correlations. The model implies that the welfare cost of inflation is nonlinear in the level of inflation and is amplified by the presence of aggregate shocks.

Keywords: money; search; inflation; unemployment; unemployment volatility; fundamental surplus; product-labor market interaction.

JEL classification: E24, E30, E40, E50.

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1 Introduction

The distortionary effects of inflation in the long run are well known. Inflation acts as a tax on cash-intensive activity, leading to reduced output, and, in the presence of labor market frictions, increased unemployment.[1] However, research on the long-run welfare cost of inflation has focused predominantly on its effect on average outcomes, largely abstracting from whether or how it affects the business cycle. In this paper, we argue that long-run inflation has significant effects on short-run unemployment volatility; more generally, the nonlinear and state-dependent effects of inflation should be taken into account when evaluating its effects on employment, output and welfare. We motivate this empirically, using cross-country data, and show the importance of nonlinearities quantitatively, using a standard monetary search framework with goods and labor market frictions.

We first use cross-country panel data from the Organization for Economic Cooperation and Development (OECD) to document three empirical correlations. First, as already expected from the existing literature and confirmed e.g. by Berentsen et al. (2011) in US data, the long-run correlation between anticipated inflation and unemployment is positive.[2] Second, there is also a positive correlation between anticipated inflation and unemployment volatility. Third, the relationship between inflation and the level of unemployment is also state-dependent and nonlinear: the positive correlation is stronger when unemployment is higher.

We then show that these correlations can be rationalized in a standard monetary search model. Our model environment closely follows the model of Berentsen et al. (2011), which combines labor and goods market frictions. Firms hire workers in a frictional labor market, and then sell some of their production to other workers in a frictional goods market, in which money is essential. We extend this framework by introducing both monetary and productivity shocks, so that our model allows for the analysis of unemployment volatility in response to these shocks, as well as the interaction between them. The insights from the analysis of unemployment volatility in the Diamond-Mortensen-Pissarides framework (Shimer (2005), Hagedorn and Manovskii (2008), Ljungqvist and Sargent (2017)) carry over naturally to our environment. The key insight is that a small surplus from an employment match amplifies the responsiveness of vacancy creation, and hence unemployment, to shocks. This well-known result turns out to have a number of novel implications once both productivity shocks and monetary shocks are present. By reducing monetary trade in the goods market, inflation lowers the surplus from a job match, thus making unemployment more responsive to productivity shocks and amplifying unemployment volatility.

[1] This is a standard result both in frameworks introducing money in reduced form, such as Cooley and Hansen (1989) and Lucas (2000), and in the monetary search literature explicitly modeling money as a medium of exchange, as in Lagos and Wright (2005) and Berentsen et al. (2011).

[2] Throughout, we use long-term nominal interest rates as a proxy for anticipated inflation; see the discussion in section 3.

Through the same channel, a higher level of inflation also makes unemployment more sensitive to further increases in inflation, implying a nonlinear effect of inflation on unemployment levels. Conversely, a decrease in productivity likewise makes unemployment more sensitive to changes in inflation. This two-way interaction between real and monetary shocks has the potential to rationalize the empirical patterns described above and generate significant nonlinearities.

Theoretical analysis of these channels is complicated by several model features. First, the magnitudes of the aforementioned amplification effects depend on the relative size of the monetary sector, which itself depends on the inflation rate, among other factors. Second, goods market frictions also lead to additional feedback effects; e.g. the households' money demand depends on the probability of meeting a seller, which in turn depends on the employment level. Third, the magnitude of the above amplification effects cannot be assessed from simple comparative statics in steady state, precisely because of the highly nonlinear behavior of the fully stochastic model. To assess the model's performance as well as its implications for volatility and welfare, we therefore simulate the model numerically. We discipline the model by calibrating parameters to match salient features of both money demand data, such as velocity, and labor market data, such as labor market flows and unemployment volatility.

We validate the model based on its ability to replicate the correlation between inflation and unemployment volatility, as well as the nonlinear correlation between inflation and unemployment levels that we find in the data. We find that the model matches these regularities well. We then illustrate the nonlinear and state-dependent dynamics of the model by computing generalized impulse response functions following a negative productivity shock. The response of employment, output, and monetary trade to shocks is stronger when trend inflation is high. For example, the average increase in unemployment on impact, in response to a one standard-deviation productivity shock, is 1.8 times stronger when trend inflation is 8% than when it is 3%.

Finally, we use the model to evaluate the welfare cost of inflation and the extent to which its interaction with volatility matters. We find that increasing the trend inflation rate from the Friedman rule to 10% leads to a welfare loss of 4.52%. Decomposing this figure reveals that the welfare cost is nonlinear in the level of inflation. In particular, increasing inflation from 0% to 5% reduces welfare by 1.76 percentage points, while increasing it from 5% to 10% leads to an additional welfare loss of 2.39 percentage points. We then analyse the contribution of aggregate uncertainty to the cost of inflation. An identical economy without aggregate shocks implies a welfare loss of only 4.26%, compared to 4.52% in the baseline economy. This amplification effect matters only at high levels of inflation. Our results thus suggest that the interaction of high inflation and aggregate uncertainty is important for assessing welfare effects – both because inflation amplifies the responsiveness of unemployment to further increases in inflation, and because inflation amplifies unemployment volatility in response to productivity shocks.

2 Relationship to literature

Our paper contributes to the research on the welfare cost of inflation in micro-founded models of money demand, starting with the work by Lagos and Wright (2005). Our work complements a growing literature combining goods and labor market frictions to study the effects of monetary policy on unemployment, starting with Berentsen et al. (2011) and developed in work such as Gomis-Porqueras et al. (2013), Rocheteau and Rodriguez-Lopez (2014), Bethune et al. (2015), Gu et al. (2021), Bethune and Rocheteau (2017), Dong and Xiao (2019), Ait Lahcen (2020), Gomis-Porqueras et al. (2020), He and Zhang (2020), and Jung and Pyun (2020), among others. As mentioned above, this literature has largely viewed short-run volatility as orthogonal to the long-run welfare effect of anticipated inflation. Perhaps this view stems from the conclusion of Cooley and Hansen (1989) that, in a *frictionless* environment, inflation has no first-order effect on the business cycle. Our findings imply that this view is not innocuous and that this conclusion is overturned in a frictional economy. To our knowledge, this is the first paper to analyze the effects of inflation in a fully stochastic model with both real and monetary shocks, and both goods and labor market frictions. There is, of course, a rich literature studying the short-run effects of monetary policy in the presence of nominal rigidities. We deliberately abstract completely from any consideration of nominal rigidities, since we want to demonstrate that inflation matters for volatility, and its effect on volatility matters for welfare, in a fully flexible-price economy.

Our paper also contributes to the research on unemployment volatility, by examining how goods market frictions and monetary trade impact unemployment fluctuations. In fact, the analysis in this paper directly builds on the well-known insight from that literature that a small job match surplus leads to high unemployment volatility (Shimer (2005), Hagedorn and Manovskii (2008), Ljungqvist and Sargent (2017)). To our knowledge, we are the first to draw implications for the effect of the inflation tax. The highly nonlinear behavior of our stochastic model is also directly related to similar findings in Petrosky-Nadeau and Zhang (2017, 2020), and Petrosky-Nadeau et al. (2018). Like them, we employ a global solution method in order to accurately characterize the model-implied dynamics. More broadly, the interaction of goods and labor market frictions also connects our work to the research by Petrosky-Nadeau and Wasmer (2015), who likewise find that such interaction has important effects for the resulting dynamics, though trade is non-monetary in these frameworks and hence they do not speak to the effects of the inflation tax.

3 Empirical evidence

In this section we present some suggestive cross-country evidence concerning the long-run relationship between anticipated inflation, unemployment, and unemployment volatility. We make no claims of causality here: the correlations we display are meant to be illustrative and serve as motivating evidence for the mechanism we highlight. We will also verify, in our numerical analysis, that our calibrated model can reproduce the correlations found here.

We use quarterly data from the Main Economic Indicators database published by the OECD, covering 35 developed countries. The start date of the period covered varies between countries and ends in 2019Q4. On the labor market side, we use the harmonised unemployment rate series in order to ensure that the data are consistent across countries. We use the long-term nominal interest rate as our proxy for the opportunity cost of holding nominal balances; this series consists mostly of yields on government bonds with a 10-year maturity. Theoretically the measure relevant for our mechanism is anticipated inflation, rather than realized inflation; hence our focus on the long-term nominal interest rate. We extract the trend component of each data series using the HP filter (Hodrick and Prescott, 1997) with a smoothing parameter value of 1600. Appendix A shows that using the 5-year moving average, instead of the HP filter, yields very similar results.

3.1 A positive relationship between $\bar{u}$ and $\bar{\iota}$

We first regress the trend component of unemployment on the trend component of the long-term nominal interest rate. Table 1 shows a positive and significant relationship. In particular, column (1) presents the results of the pooled OLS regression

$$\bar{u}_{jt} = \alpha + \beta \bar{\iota}_{jt} + \varepsilon_{jt}, \tag{1}$$

where $\bar{u}_{jt}$ and $\bar{\iota}_{jt}$ represent the trend components of unemployment and the long-term nominal interest rate in country j at quarter t. It indicates that a 1 percentage point (pp) increase in $\bar{\iota}$ is associated with a 0.351pp increase in $\bar{u}$. Column (2), (3) and (4) present results from fixed-effects panel regressions of the type

$$\bar{u}_{jt} = \alpha + \beta \bar{\iota}_{jt} + \gamma_j + \delta_t + \varepsilon_{jt}, \tag{2}$$

where γ_j and δ_t represent, respectively, country and time fixed effects. The results of all three panel regressions confirm the positive relationship between $\bar{u}$ and $\bar{\iota}$. In particular, the estimates presented in column (4), which include both country and time fixed effects, indicate that a 1pp increase in $\bar{\iota}$ is associated with a 0.772pp increase in $\bar{u}$, an economically and statistically significant relationship.

To verify the robustness of our finding, table 7 in Appendix A reports the results of running the same regression specifications on trend components of unemployment and the long-term nominal interest rate extracted using a 5-year moving average instead of the HP filter. The results are similarly significant and strong. In particular, the specification in column (4), which includes both country and time fixed effects, indicates that a 1pp increase in $\bar{\iota}$ is associated with a 0.915pp increase in $\bar{u}$. Finally, tables 8 and 9 in Appendix A present the regression results using the

Table 1: Regression of $\bar{u}$ on $\bar{\imath}$

	Trend unemployment			
	(1)	(2)	(3)	(4)
Constant	5.771***	6.036***	3.739***	3.498***
	(0.618)	(0.362)	(1.366)	(1.209)
Trend long-term rate	0.351***	0.301***	0.727**	0.772***
	(0.091)	(0.062)	(0.288)	(0.224)
Observations	4,007	4,007	4,007	4,007
R^2	0.086	0.140	0.121	0.135
F-Statistic	377.98***	646.61***	515.56***	581.55***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parentheses. Data are from the OECD. Both unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

logarithm of unemployment instead of its level with HP and 5-year moving average trends; the results are again very similar.[3] These cross-country regression results are thus consistent with the positive long-run relationship between unemployment and inflation in the US, documented by Beyer and Farmer (2007), Berentsen et al. (2011) and Haug and King (2014).

3.2 A positive relationship between unemployment volatility and $\bar{\imath}$

Next, we seek to provide some evidence on the relationship between anticipated inflation and unemployment volatility. We use the HP-filtered cyclical component of the logarithm of unemployment. Our measure of volatility $\sigma_{u_{jt}}$ is the 5-year rolling window standard deviation of this cyclical component. Column (1) of table 2 presents the results of the pooled OLS regression

$$\sigma_{u_{jt}} = \alpha + \beta \bar{\imath}_{jt} + \varepsilon_{jt}, \tag{3}$$

[3]In addition, we run similar panel regressions using either the unfiltered or the cyclical components of unemployment and the short-term nominal interest rate. The resulting coefficient is either statistically not significant (for the unfiltered data) or significant and slightly negative (for the HP-detrended data). These results are available on request.

Table 2: Regression of unemployment volatility on $\bar{\iota}$

	log unemployment volatility			
	(1)	(2)	(3)	(4)
Constant	0.058***	0.052***	0.060***	0.031
	(0.005)	(0.007)	(0.011)	(0.023)
Trend long-term rate	0.005***	0.006***	0.005*	0.010**
	(0.001)	(0.001)	(0.002)	(0.004)
Observations	3,616	3,616	3,616	3,616
R^2	0.079	0.115	0.031	0.062
F-Statistic	310.07***	463.69***	109.18***	221.79***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Unemployment volatility is measured as the 5-year rolling window standard deviation of HP-detrended log unemployment. Long-term nominal interest rate series for each country is filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

while columns (2), (3) and (4) present the results of the fixed effects panel regression

$$\sigma_{u_{jt}} = \alpha + \beta\bar{\iota}_{jt} + \gamma_j + \delta_t + \varepsilon_{jt}, \tag{4}$$

with country and/or time fixed effects. Our preferred specification in column (4) shows that an increase of 1pp in $\bar{\iota}$ is associated with a 0.01 increase in the volatility of unemployment, which corresponds to about 12% of average volatility. Table 10 in Appendix A presents very similar results obtained by running the same panel regressions using the level of unemployment instead of the logarithm of unemployment.

In Appendix A, we also compute unemployment volatility based on the cyclical component of the logarithm of unemployment extracted assuming a 5-year moving average as the trend, and then regress it on the 5-year moving average of the long-run interest rate. Table 11 presents the results. Column (4) with country and time fixed effects shows that an increase of 1pp in trend ι is associated with a 0.026pp increase in unemployment volatility. This increase corresponds to about 18% of average unemployment volatility. We also run the same panel regressions using the level of unemployment and the results, presented in table 12 in Appendix A, are very similar.

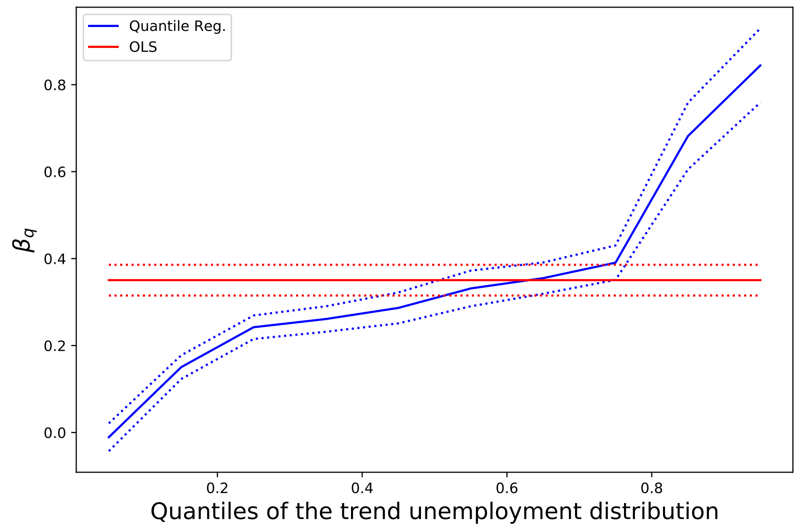


Figure 1: Quantile regression coefficients of $\bar{u}$ on $\bar{\imath}$ for various quantiles of $\bar{u}$.
Notes: The dashed lines represent the 95% confidence intervals. Data are from the OECD. Both unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

3.3 A state-dependent relationship between $\bar{u}$ and $\bar{\imath}$

Finally, we want to examine whether the positive long-run relationship observed between unemployment and anticipated inflation varies with the level of unemployment. To do so, we use a quantile regression specification. As opposed to the linear regression, which approximates the conditional expectation function, the quantile regression approximates the conditional quantile function at quantile q by the linear relationship

$$\mathcal{Q}_q(\bar{u}_{jt}|\bar{\imath}_{jt}) = \alpha_q + \beta_q \bar{\imath}_{jt} + \varepsilon_{qjt}. \tag{5}$$

By estimating the above regression for different quantiles of the distribution of $\bar{u}$, we can check whether the relationship between the low frequency components of unemployment and nominal interest rates varies with the level of unemployment. Figure 1 plots, in solid blue, the values of β_q for various values of q estimated by pooling together all observations in our sample, with the dashed blue lines depicting the 95% confidence interval. The coefficient of the pooled OLS regression discussed above is depicted as the flat red line for comparison. The relationship is clearly nonlinear. At the 5th percentile of the distribution of trend unemployment, i.e. $q = 5\%$, a 1pp increase in the trend long-term nominal interest rate is associated with almost no change in trend unemployment, whereas at the 95th percentile the coefficient reflects an increase of 0.84pp in trend unemployment. The quantile regression depicted in figure 8 in Appendix A uses 5-year moving averages and the ones depicted in figures 9 and 10 use the logarithm of unemployment with 5-year moving averages and HP-filtered trends, respectively. The results are very similar.

To summarize, we have provided illustrative cross-country evidence that there is a positive long-run correlation between anticipated inflation, as proxied by nominal interest rates, and unemployment; that there is also a positive correlation between anticipated inflation and unemployment volatility; and that the inflation-unemployment correlation is stronger at high unemployment rates. The rest of the paper explores the mechanisms through which a monetary search model can generate these patterns and the implications of these mechanisms.

4 Model

The model environment closely follows Berentsen et al. (2011), augmented with aggregate productivity and monetary shocks. Time is discrete, and the time horizon is infinite. The economy is populated by a continuum of infinitely-lived households and a continuum of infinitely-lived firms. Households have preferences

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[h_t + \mathrm{u}\left(x_t\right) \right], \tag{6}$$

where h_t is consumption of a general good (taken as the numeraire), and x_t is consumption of a specialized good. Firms consume the general good and have utility

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t h_t. \tag{7}$$

Each period is divided into three sub-periods. In the first sub-period (LM), there is a frictional labor market, in which households and firms match pairwise to produce the specialized good. In the second sub-period (DM), there is a frictional goods market, in which productive firms sell the specialized good to other households. In the third sub-period (CM), a frictionless centralized market convenes, in which firms pay wages and liquidate unsold inventories, and all agents rebalance their portfolios.

Matching and production (LM). At each point in time, each household is either employed or unemployed. Matching of unemployed households (workers) and firms (employers) in the LM is random. Firms create vacancies at cost κ. If the measure of vacancies is v_t and the measure of unemployed households at the beginning of period t is $1 - n_{t-1}$, the measure of new matches created in period t is $\mathcal{M}\left(v_t, 1 - n_{t-1}\right)$, where $\mathcal{M}$ is a constant returns to scale function satisfying the standard assumptions. The job-finding probability for an unemployed household is then

$$\frac{\mathcal{M}\left(v_t, 1-n_{t-1}\right)}{1-n_{t-1}} = \mathcal{M}\left(\theta_t, 1\right) \equiv f\left(\theta_t\right), \tag{8}$$

where $\theta_t = v_t / \left(1-n_{t-1}\right)$ is the market tightness. Similarly, the probability for a firm of filling its vacancy is

$$\frac{\mathcal{M}\left(v_t, 1-n_{t-1}\right)}{v_t} = \mathcal{M}\left(1, \frac{1}{\theta_t}\right) \equiv q\left(\theta_t\right). \tag{9}$$

Existing matches are destroyed every period with an exogenous probability δ. The above implies that the employment level upon exiting the period-t LM is

$$n_t = \left(1-\delta\right) n_{t-1} + f\left(\theta_t\right)\left(1-n_{t-1}\right). \tag{10}$$

A job match between a household and a firm produces y_t units of the general good in the LM, to be sold in the subsequent DM (see below). The productivity of a match, y_t, is subject to aggregate shocks and follows an AR(1) process

$$\log y_t = \left(1-\rho_y\right)\log \bar{y} + \rho_y \log y_{t-1} + \sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim \mathcal{N}\left(0,1\right). \tag{11}$$

Trade in the goods market (DM). The DM is a decentralized goods market, in which productive firms (i.e. firms who have a worker in the LM) match pairwise with households in order to convert their general good into specialized goods and sell them. A firm with a worker in the LM has a technology for converting general goods into specialized goods: producing x_t units of the specialized good costs $\mathrm{c}\left(x_t\right)$ units of the general good. Matching is random and governed by a constant returns to scale matching function. Specifically, since there are n_t productive firms seeking to sell and a measure 1 of households seeking to buy, each household meets a firm with probability $\alpha\left(n_t\right)$, and a firm meets a household with probability $\alpha\left(n_t\right)/n_t$, where $\alpha' > 0$, $\alpha'' < 0$, and $\alpha\left(n\right) \leq n$. Terms of trade between a firm and a household in the DM are determined by proportional (Kalai) bargaining and are described in detail below.

Meetings in the DM are anonymous, there is no record-keeping of past transactions, and agents cannot commit to repay loans. This rules out credit in the DM, making a medium of exchange necessary for trade. This role is served by fiat money. The supply of fiat money is augmented stochastically via lump-sum transfers T_t to households in the CM. We find it convenient to think of monetary policy in terms of the nominal interest rate $\iota_t = \left(1+\pi_t\right)/\beta - 1$, where π_t is the rate of inflation. When we refer, below, to the Friedman rule, we mean a policy where $\iota = 0$.

Settlement and rebalancing (CM). The CM is a frictionless market, in which wages are paid out to employed households, and agents decide on money holdings for the following period. Wages

w_t to be paid out in the CM are determined in the previous LM by Nash bargaining between the household and the firm (see below). Unemployed households receive an exogenous amount b of the general good, which can be interpreted as unemployment benefits, a value of leisure, or home production.

5 Equilibrium

We define equilibrium recursively. The aggregate state of the economy is $\Omega = (n^-, \iota, y)$, where n^- is the start-of-period employment level, ι is the nominal interest rate, and y is productivity. Throughout, "$^-$" superscripts are used to denote the previous period's variables, and "$^+$" superscripts are used to denote the next period's variables.

The timing is as follows. The ι and y shocks realize at the beginning of the CM. The realized ι shock determines the amount of lump-sum transfers $T(\iota)$ that will be paid in the following CM. Agents then make portfolio choices, and at the same time firms post vacancies v at cost κ. In the subsequent LM, matching and separations take place. The market tightness is

$$\theta = \frac{v}{1 - n^-} \tag{12}$$

and the law of motion for employment is

$$n = (1 - \delta) n^- + f(\theta) \left(1 - n^-\right). \tag{13}$$

Wages are negotiated at this stage. After that, worker-firm matches produce and go on to the DM, where they sell their output in exchange for money. The buyer's matching probability in the DM is then $\alpha(n)$. A firm with productivity y selling x units of output incurs a cost of $\mathrm{c}(x)$. A firm who did not trade receives y units of output. Finally, in the CM either wages (w) or unemployment benefits (b) are paid, along with transfers $T(\iota)$.

5.1 Workers

If a worker enters the CM with employment status $j \in \{e, u\}$ and asset position a (in real terms), their value function is

$$V_{CM}^j(a, \Omega) = \max_{c,z} c + \beta V_{LM}^j(z, \Omega), \tag{14}$$

subject to the budget constraint

$$c + (1 + \pi) z = a. \tag{15}$$

The outcome of this maximization problem gives us the real balances policy function $z(\Omega)$, which, by quasi-linearity, is independent of a.

Now, consider the LM. An employed worker ($j = e$) stays employed with probability $1 - \delta$. An unemployed worker ($j = u$) finds a job with probability $f(\theta)$, where the dependence of $\theta = \theta(\Omega)$ on Ω is understood.

$$V^e_{LM}(z, \Omega) = (1 - \delta) V^e_{DM}(z, w(\Omega), \Omega) + \delta V^u_{DM}(z, \Omega), \tag{16}$$

$$V^u_{LM}(z, \Omega) = f(\theta) V^e_{DM}(z, w(\Omega), \Omega) + (1 - f(\theta)) V^u_{DM}(z, \Omega). \tag{17}$$

Next, consider the DM. Each worker meets a firm in the product market with probability $\alpha(n)$, where $n = n(\Omega)$ is the end-of-period employment level as given by (13). If meeting a firm, he buys some negotiated amount x of the special good in exchange for some negotiated amount $d \leq z$ of real balances. As we will verify below, x, d (determined by bargaining) depend on z but not on the worker's employment status, and will therefore be written as $\tilde{x}(z, \Omega)$, $\tilde{d}(z, \Omega)$. Thus:

$$\begin{aligned} V^e_{DM}(z, w, \Omega) =& \alpha(n) \left[\mathrm{u}(\tilde{x}(z, \Omega)) + \mathbb{E} V^e_{CM}\left(z - \tilde{d}(z, \Omega) + T(\iota) + w, \Omega^+\right)\right] \\ &+ (1 - \alpha(n)) \mathbb{E} V^e_{CM}\left(z + T(\iota) + w, \Omega^+\right) \end{aligned}, \tag{18}$$

$$\begin{aligned} V^u_{DM}(z, \Omega) =& \alpha(n) \left[\mathrm{u}(\tilde{x}(z, \Omega)) + \mathbb{E} V^u_{CM}\left(z - \tilde{d}(z, \Omega) + T(\iota) + b, \Omega^+\right)\right] \\ &+ (1 - \alpha(n)) \mathbb{E} V^u_{CM}\left(z + T(\iota) + b, \Omega^+\right) \end{aligned}, \tag{19}$$

where Ω^+ denotes the following period's aggregate state.

5.2 Firms

In the CM, a firm with a worker consumes its unsold output o net of the wage w:

$$J^e_{CM}(o, w, \Omega) = o - w + \beta J^e_{LM}(\Omega). \tag{20}$$

A firm without a worker decides whether to post a vacancy, at cost κ:

$$J^v_{CM}(\Omega) = \max\{0, -\kappa + \beta J^v_{LM}(\Omega)\}. \tag{21}$$

In the LM, a firm with a worker loses that worker with probability δ, and a firm with a vacancy fills it with probability $q(\theta)$:

$$J^e_{LM}(\Omega) = (1 - \delta) J^e_{DM}(w(\Omega), \Omega) + \delta J^v_{DM}(\Omega), \tag{22}$$

$$J^v_{LM}(\Omega) = q(\theta) J^e_{DM}(w(\Omega), \Omega) + (1 - q(\theta)) J^v_{DM}(\Omega). \tag{23}$$

In the DM, a firm with a worker produces and sells its output, getting

$$\begin{aligned} J^e_{DM}(w,\Omega) = & \frac{\alpha(n)}{n}\mathbb{E}J^e_{CM}\left(y - \mathrm{c}\left(x\left(\Omega\right)\right) + d\left(\Omega\right), w, \Omega^+\right) \\ & + \left(1 - \frac{\alpha(n)}{n}\right)\mathbb{E}J^e_{CM}\left(y, w, \Omega^+\right) \end{aligned}, \tag{24}$$

where the traded quantities $x(\Omega)$, $d(\Omega)$ are determined by bargaining (as described below) through

$$x(\Omega) = \tilde{x}(z(\Omega), \Omega),$$

$$d(\Omega) = \tilde{d}(z(\Omega), \Omega).$$

In other words, the firm takes as given the z of the worker it will meet when forecasting the traded quantities. A firm without a worker does not trade in the DM, so

$$J^v_{DM}(\Omega) = \mathbb{E}J^v_{CM}\left(\Omega^+\right). \tag{25}$$

5.3 Goods market bargaining

We now turn to the determination of x, d given z. Consider a worker with employment status j, real balances z, and other income a, meeting a firm with promised wage w. Then the traded amounts $x = \tilde{x}(z, \Omega)$, $d = \tilde{d}(z, \Omega)$ for such a worker are determined by

$$\max_{x,d} \mathrm{u}(x) + \mathbb{E}\left[V^j_{CM}\left(z - d + a, \Omega^+\right) - V^j_{CM}\left(z + a, \Omega^+\right)\right], \tag{26}$$

subject to $d \leq z$ and

$$\begin{aligned} \mathrm{u}(x) + & \mathbb{E}\left[V^j_{CM}\left(z - d + a, \Omega^+\right) - V^j_{CM}\left(z + a, \Omega^+\right)\right] \\ & = \frac{\varphi}{1-\varphi}\mathbb{E}\left[J^e_{CM}\left(y - \mathrm{c}(x) + d, w, \Omega^+\right) - J^e_{CM}\left(y, w, \Omega^+\right)\right]. \end{aligned} \tag{27}$$

Since V^j_{CM} is linear in its first argument, and the marginal value of z does not depend on j or Ω^+, we can write, for any a,

$$\mathrm{u}(x) + \mathbb{E}\left[V^j_{CM}\left(z - d + a, \Omega^+\right) - V^j_{CM}\left(z + a, \Omega^+\right)\right] = \mathrm{u}(x) - d. \tag{28}$$

On the firm's side, we can similarly write

$$\mathbb{E}\left[J^e_{CM}\left(y - \mathrm{c}(x) + d, w, \Omega^+\right) - J^e_{CM}\left(y, w, \Omega^+\right)\right] = d - \mathrm{c}(x). \tag{29}$$

This means that we can re-write the bargaining problem as

$$\max_{x,d} \mathrm{u}(x) - d \quad s.t. \quad \mathrm{u}(x) - d = \frac{\varphi}{1-\varphi}\left[d - \mathrm{c}(x)\right] \quad \text{and} \quad d \leq z. \tag{30}$$

The solution is well known. Define $\mathrm{g}(x) = (1-\varphi)\,\mathrm{u}(x) + \varphi \mathrm{c}(x)$, and define x^* as the solution to $\mathrm{u}'(x) = \mathrm{c}'(x)$. The solution to the bargaining problem is

$$\begin{aligned} x &= \min\left\{x^*, \mathrm{g}^{-1}(z)\right\}, \\ d &= \min\left\{\mathrm{g}(x^*), z\right\}. \end{aligned} \tag{31}$$

5.4 Optimal choice of real balances

The DM bargaining solution described above gives us

$$\frac{\partial V_{LM}^j}{\partial z} = 1 + \alpha(n) \max\left\{0, \frac{\mathrm{u}'(x)}{\mathrm{g}'(x)} - 1\right\}. \tag{32}$$

In the CM, the first-order condition for z is

$$1 + \iota = \frac{\partial V_{LM}^j}{\partial z}. \tag{33}$$

Combining these, we get

$$\mathrm{u}'(x) = \left(1 + \frac{\iota}{\alpha(n)}\right) \mathrm{g}'(x) \tag{34}$$

and $z = \mathrm{g}(x)$.

5.5 Labor market bargaining

We next consider the determination of the wage. The worker's surplus from being employed at wage w is

$$V_{DM}^e(z, w, \Omega) - V_{DM}^u(z, \Omega) = V_{DM}^e(0, w, \Omega) - V_{DM}^u(0, \Omega) \equiv S_{DM}^e(w, \Omega).$$

This can be written recursively as

$$S_{DM}^e(w, \Omega) = w - b + \beta \mathbb{E}\left(1 - \delta - f\left(\theta\left(\Omega^+\right)\right)\right) S_{DM}^e\left(w\left(\Omega^+\right), \Omega^+\right). \tag{35}$$

The firm's surplus from having a worker at wage w is $J^e_{DM}(w,\Omega)$. Defining the firm's total output as

$$\begin{aligned}\mathcal{O}(\Omega) =& y + \frac{\alpha(n)}{n}\left(y + d(\Omega) - \mathrm{c}(x(\Omega)) - y\right) \\ =& y + \frac{\alpha(n)}{n}(1-\varphi)\left(\mathrm{u}(x(\Omega)) - \mathrm{c}(x(\Omega))\right),\end{aligned} \tag{36}$$

we can write

$$J^e_{DM}(w,\Omega) = \mathcal{O}(\Omega) - w + \beta(1-\delta)\mathbb{E}J^e_{DM}\left(w\left(\Omega^+\right),\Omega^+\right). \tag{37}$$

The surplus from an employment match is $\mathcal{S}(\Omega) = S^e_{DM}(w,\Omega) + J^e_{DM}(w,\Omega)$. We assume that the wage is determined by Nash bargaining, with worker bargaining weight equal to ξ, so that $w = w(\Omega)$ solves

$$S^e_{DM}(w(\Omega),\Omega) = \xi\mathcal{S}(\Omega). \tag{38}$$

Adding (35) and (37) and using (38), we obtain

$$\mathcal{S}(\Omega) = \mathcal{O}(\Omega) - b + \beta\mathbb{E}\left(1-\delta-\xi f\left(\theta\left(\Omega^+\right)\right)\right)\mathcal{S}\left(\Omega^+\right). \tag{39}$$

5.6 Free entry and wages

Market tightness $\theta = \theta(\Omega)$ is determined by the free entry condition $\kappa = \beta J^v_{LM}(\Omega)$, which implies

$$\kappa = \beta q(\theta)(1-\xi)\mathcal{S}(\Omega). \tag{40}$$

Combining (40) with the expression for the match surplus in (39), we also obtain the expression for the wage,

$$w(\Omega) = \xi\mathcal{O}(\Omega) + (1-\xi)b + \mathbb{E}\xi\kappa\theta\left(\Omega^+\right). \tag{41}$$

5.7 Equilibrium conditions

The equilibrium consists of functions $x(\Omega)$, $\mathcal{O}(\Omega)$, $\mathcal{S}(\Omega)$, $\theta(\Omega)$, $w(\Omega)$, and $n(\Omega)$ satisfying the law of motion (13), optimal choice of real balances (34), the output equation (36), the Bellman equation for the match surplus (39), the free entry condition (40), and the wage equation (41).

6 Mechanism

As in any environment building on the Diamond-Mortensen-Pissarides framework, the volatility of market tightness and therefore employment depends on the size of the match surplus. Moreover, amplification in our model, relative to the benchmark DMP model, is enriched by two features. First, there are two shocks – productivity and monetary – which interact with one another, e.g. inflation affects the size of the employment response to productivity, and vice versa. Second, goods market frictions and the endogenous choice of real balances mean that the output of a match is endogenous, introducing additional feedback effects through its dependency on employment.

To understand the main mechanisms behind the model's dynamics, we briefly consider the comparative statics of the model in steady state, similarly to Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2017). In steady state, the free entry condition (40) becomes

$$\kappa = \underbrace{\frac{\beta q(\theta)(1-\xi)}{1-\beta(1-\delta-\xi f(\theta))}}_{\Upsilon(\theta)} (\mathcal{O} - b). \tag{42}$$

The steady-state employment level is then given by

$$n = \frac{f(\theta)}{\delta + f(\theta)}. \tag{43}$$

Equation (42) implicitly defines steady-state θ, and therefore n, as a function of $\mathcal{O}$. In turn, from (34) and (36), $\mathcal{O}$ is a function of n, ι and y.

6.1 The match surplus channel

We first discuss how the Diamond-Mortensen-Pissarides component of our model leads to amplification of shocks through a standard match surplus channel. Using (42), we can write the elasticity of market tightness with respect to y,

$$\varepsilon_{\theta,y} \equiv \frac{y}{\theta}\frac{\partial\theta}{\partial y} = \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\varepsilon_{\mathcal{O},y}, \qquad \varepsilon_{\mathcal{O},y} \equiv \frac{y}{\mathcal{O}}\frac{\partial\mathcal{O}}{\partial y}, \tag{44}$$

and the semi-elasticity of market tightness with respect to ι,

$$\varepsilon_{\theta,\iota} \equiv \frac{1}{\theta}\frac{\partial\theta}{\partial\iota} = \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\varepsilon_{\mathcal{O},\iota}, \qquad \varepsilon_{\mathcal{O},\iota} \equiv \frac{1}{\mathcal{O}}\frac{\partial\mathcal{O}}{\partial\iota}, \tag{45}$$

where $\epsilon_{\Upsilon,\theta} = -\theta\Upsilon'(\theta)/\Upsilon(\theta)$, with Υ defined in (42). We adopt the convention of writing $\varepsilon_{\theta,\iota}$ and $\varepsilon_{\mathcal{O},\iota}$ as semi-elasticities since ι is typically written in percentage terms.

The message from (44) and (45) is that the sensitivity of employment to shocks affecting match output depends on a component related to the matching function, $\epsilon_{\Upsilon,\theta}$, and the relative size of b and $\mathcal{O}$: the lower the net surplus from a match, the more elastic this surplus is. If the productivity of a match were exogenous (as would be the case in a non-monetary model with $\mathcal{O} = y$), this would reduce to the standard small-surplus argument, see e.g. Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2017). Here, of course, with endogenous output, one must account for further feedback effects.

6.2 Decomposition of match surplus and feedback effects

The monetary-search dimension of our model implies that match output itself depends on agents' money demand and goods market frictions, as captured by the terms $\varepsilon_{\mathcal{O},y}$ and $\varepsilon_{\mathcal{O},\iota}$. Note that the steady-state version of (36) gives

$$\begin{aligned}\mathcal{O} =& y + \frac{\alpha(n)}{n}(1-\varphi)(\mathrm{u}(x) - \mathrm{c}(x)) \\ \equiv& y + \mathcal{P},\end{aligned} \tag{46}$$

where x is the steady-state DM quantity traded, and $\mathcal{P}$ denotes the DM output in units of the numeraire. In turn, x is determined by the optimality condition (34) for real balances in steady state, which can be rearranged as

$$\varphi\frac{\mathrm{c}'(x)}{\mathrm{u}'(x)} = \frac{\alpha(n)}{\iota + \alpha(n)} - (1-\varphi). \tag{47}$$

Differentiating (46) and (47) totally with respect to y and substituting into (44), we derive in Appendix C that

$$\varepsilon_{\theta,y} = \left(1 - \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\frac{\mathcal{P}}{\mathcal{O}}\varepsilon_{\mathcal{P},n}\varepsilon_{n,\theta}\right)^{-1}\frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\frac{y}{\mathcal{O}}, \tag{48}$$

where $\varepsilon_{\mathcal{P},n}$ is the elasticity of the endogenous part of output to employment given by

$$\varepsilon_{\mathcal{P},n} = \frac{1}{\sigma_{u,x} + \sigma_{c,x}}\frac{\iota\alpha(n)}{\iota + \alpha(n)}\frac{\epsilon_{\alpha,n}}{\varphi\alpha(n) - (1-\varphi)\iota}\frac{x(\mathrm{u}'(x) - \mathrm{c}'(x))}{\mathrm{u}(x) - \mathrm{c}(x)} - (1-\epsilon_{\alpha,n}), \tag{49}$$

with $\epsilon_{\alpha,n} = n\alpha'(n)/\alpha(n)$; $\varepsilon_{n,\theta} = \frac{\theta}{n}\frac{dn}{d\theta}$ derived from (43); and $\sigma_{u,x} = -x\mathrm{u}''(x)/\mathrm{u}'(x)$, $\sigma_{c,x} = x\mathrm{c}''(x)/\mathrm{c}'(x)$.

Equations (48) and (49) illustrate the model's amplification mechanism in response to shocks to y. A shock to y has a direct effect on $\mathcal{O}$ and therefore on match surplus; this direct effect discourages vacancy creation, more so when $y/(\mathcal{O}-b)$ is large. However, our model also contains a feedback effect, captured by term in parentheses in (48), through which y also affects $\mathcal{P}$. Because a higher y raises match surplus, it raises employment, which in turn raises the buyers' probability of finding a seller in the DM, driving up the demand for real balances and thus the profitability of a match for the firm. But higher employment may also reduce the probability that the firm finds a match with the buyer, which would reduce the expected profitability from operating in the DM. The sign of $\varepsilon_{\mathcal{P},n}$ is therefore theoretically ambiguous.[4]

We can apply a similar strategy to derive and interpret $\varepsilon_{\theta,\iota}$. Differentiating (46) and (47) fully with respect to ι and substituting into (45), we derive in Appendix C that

$$\varepsilon_{\theta,\iota} = \left(1 - \frac{1}{\epsilon_{\Upsilon,\theta}} \frac{\mathcal{O}}{\mathcal{O}-b} \frac{\mathcal{P}}{\mathcal{O}} \varepsilon_{\mathcal{P},n} \varepsilon_{n,\theta}\right)^{-1} \frac{1}{\epsilon_{\Upsilon,\theta}} \frac{\mathcal{O}}{\mathcal{O}-b} \frac{\mathcal{P}}{\mathcal{O}} \varepsilon_{\mathcal{P},\iota}, \tag{50}$$

where

$$\varepsilon_{\mathcal{P},\iota} = -\frac{1}{\sigma_{u,x} + \sigma_{c,x}} \frac{\alpha(n)}{\iota + \alpha(n)} \frac{1}{\varphi\alpha(n) - (1-\varphi)\iota} \frac{x(\mathrm{u}'(x) - \mathrm{c}'(x))}{\mathrm{u}(x) - \mathrm{c}(x)}. \tag{51}$$

A monetary shock directly lowers the match surplus by lowering buyers' demand for real balances. Moreover, this decrease in match surplus leads, all else equal, to lower employment. This generates a similar feedback effect, operating through the monetary-search frictions, as observed with a shock to y.

6.3 The effects of inflation on the amplification of shocks

We finally examine how the level of inflation (as measured by ι) affects the reaction of unemployment to productivity and monetary shocks, i.e. the elasticities $\varepsilon_{\theta,y}$ and $\varepsilon_{\theta,\iota}$. First, since a higher inflation lowers x (through (47)), it directly lowers the match output $\mathcal{O} = y + \mathcal{P}$, and hence $\frac{\mathcal{O}}{\mathcal{O}-b}$, amplifying volatility through a standard small-surplus channel. It also raises $y/\mathcal{O}$, thereby raising the fraction of the match output directly responsive to the productivity shocks, which amplifies shocks to productivity, while reducing the fraction of match output directly responsive to the monetary shock, which dampens the effects of the monetary shock.

The level of inflation also affects the feedback effect between employment and the monetary-search frictions through $\varepsilon_{\mathcal{P},n}$, detailed in (49). While the effect of ι on $\varepsilon_{\mathcal{P},n}$ is theoretically ambiguous in general, we note that $\mathrm{u}'(x) - \mathrm{c}'(x)$ is decreasing in x, and hence $\varepsilon_{\mathcal{P},n}$ is likely to be small in the neighborhood of the Friedman rule (since $\mathrm{u}'(x) = \mathrm{c}'(x)$ at the Friedman rule). Intuitively, when inflation is low, a marginal increase in inflation raises the importance of distortions in the frictional goods market for the buyers, in turn making trade in that market more sensitive to the probability that a buyer finds a match. If operational, this channel would imply that inflation further amplifies both shocks in our model.

Finally, inflation also affects $\varepsilon_{\mathcal{P},\iota}$, which matters for the propagation of the monetary shock. The sign of this effect is ambiguous, but it can be shown that inflation has an amplifying effect on

[4] In Appendix C, we argue why the term in parenthesis in (48) should nevertheless be non-negative in a locally stable equilibrium.

$\varepsilon_{\mathcal{P},\iota}$ in the neighborhood of the Friedman rule which also dominates the dampening effect caused by the reduced fraction of match output directly responsive to the monetary shock. This is evident from the fact that $\mathrm{u}'(x) - \mathrm{c}'(x)$, hence $\varepsilon_{\mathcal{P},\iota}$, is zero at the Friedman rule.

Summarizing, inflation can have an amplifying effect both on unemployment responsiveness to productivity shocks, and on unemployment responsiveness to monetary shocks, i.e. further increases in inflation. It also makes clear the necessity to assess the nonlinear effects of inflation quantitatively. We will do so in the next section by calibrating the model to match salient features of the US labor market as well as the behavior of money demand.

7 Calibration

Our calibration strategy is to match features of both labor market dynamics and monetary data in the US. As pointed out earlier, two features distinguish our environment from the standard labor search model. First, our model economy is subject to two shocks: productivity and nominal interest rates. Second, measured output per worker includes DM trade and is therefore endogenous. As a consequence, the productivity process cannot be backed out directly from output per worker as, e.g., in Shimer (2005) or Hagedorn and Manovskii (2008), and will instead be calibrated jointly with other model parameters.

The model is calibrated on a monthly basis using a mix of monthly and quarterly data. We use, whenever possible, data series covering the period from January 1948 to December 2019.[5] Except for the discount factor β, the job separation rate δ and the exogenous process of interest rate shocks, the model's remaining parameters are calibrated jointly such that a selected set of model-based simulated moments match their empirical counterparts. We set the monthly discount factor β externally to 0.998 to be consistent with an average monthly real interest rate of 0.23%.

Labor market. The exogenous process for y_t in the model is assumed to follow the AR(1) process

$$\log y_t = (1 - \rho_y) \log \bar{y} + \rho_y \log y_{t-1} + \sigma_y \varepsilon_{y,t} \tag{52}$$

where $\varepsilon_y \sim \mathcal{N}(0, 1)$ and $\bar{y}$ is normalized to 1. We calibrate the process parameters ρ_y and σ_y internally such that the cyclical component of the logarithm of total output per worker in the model, $\mathcal{O}_t$, matches the volatility and autocorrelation of output per worker in the data. We follow Shimer (2005) in using the Bureau of Labor Statistics' (BLS) monthly data series measuring real output per person in the non-farm business sector. We extract the cyclical component of the logarithm of the quarterly observations using the HP filter with a smoothing parameter value of 1600.

[5] Our sample stops short of the onset of the COVID-19 pandemic and hence does not capture the wild movements in macroeconomic variables that occurred afterward.

Similarly to Hagedorn and Manovskii (2008), we use the elasticity of wages with respect to labor productivity as a calibration target to identify the worker bargaining weight. We use the BLS series for labor productivity and labor income share to compute a series for the real wage as the product of the two. We then extract the HP-filtered cyclical component of the logarithm of the computed wage series and use it to estimate the elasticity of the wage to labor productivity. We include the wage elasticity in our list of targeted moments and add the bargaining power of workers ξ to the internal calibration to match it.

We calibrate the cost of posting vacancies κ to match the average labor market tightness θ, computed as the ratio of the vacancy and unemployment rates. The vacancy rate series is constructed as in Petrosky-Nadeau and Zhang (2020). From December 2000 to December 2019, the number of job openings is obtained from the Job Openings and Labor Turnover Survey (JOLTS) of the BLS which we divide by the civilian labor force to obtain the vacancy rate. From January 1951 to November 2000 we use the vacancy rate series from Barnichon (2010), which is based on a composite print and online help wanted index. We then splice Barnichon's series to the JOLTS series in December 2000 to obtain one series stretching from January 1951 to December 2019.

We assume the matching function

$$\mathcal{M}(v, 1-n) = \frac{v(1-n)}{(v^{\chi} + (1-n)^{\chi})^{1/\chi}}, \tag{53}$$

similarly to Den Haan et al. (2000), as it ensures job finding and vacancy filling probabilities lie in the interval $[0, 1]$. The corresponding job finding probability as a function of θ is

$$f(\theta) = \frac{\theta}{(1+\theta^{\chi})^{1/\chi}}. \tag{54}$$

We add χ to our internal calibration to match the average job finding probability, which we compute using data on short-term unemployment as in Shimer (2012).[6] We calculate the job separation rate in the standard way by dividing the number of short-term unemployed workers by last period's employed population. We obtain an average monthly separation rate of 2.51%.[7] We set δ directly to that value.

The value of non-market activity b is calibrated to match the standard deviation of unemployment, which stands at 0.138. This statistic is measured in logs as the quarterly deviation from an HP-filtered trend with a smoothing parameter of 1600.

[6]We correct for the 1994 CPS redesign following Shimer (2012).

[7]Correcting for time aggregation following Shimer (2005) yields a very similar value.

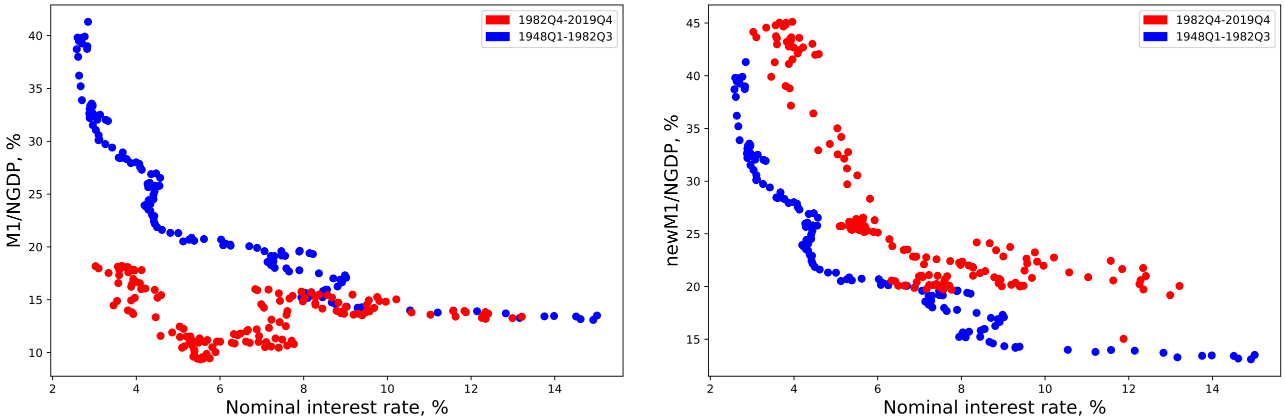


Figure 2: Measuring money demand: M1 v. NewM1.

Decentralized goods market. We assume the utility of DM good consumption takes the form

$$\mathrm{u}\left(x\right) = A\frac{x^{1-\gamma}}{1-\gamma}, \tag{55}$$

where $\gamma \in (0,1)$ and $A > 0$. For the DM cost function, we set

$$\mathrm{c}\left(x\right) = x. \tag{56}$$

For the DM matching function, we assume that the buyer's probability of finding a seller is

$$\alpha\left(n\right) = \zeta\frac{n}{1+n}, \tag{57}$$

where n is the measure of active sellers (i.e. firms with workers), 1 is the measure of active buyers (households), and ζ is a matching efficiency parameter.

Most of the parameters related to the DM are calibrated following Lagos and Wright (2005) and Berentsen et al. (2011). In particular, A and γ are calibrated to match both the average ratio of aggregate money supply to nominal GDP (i.e. the inverse of the velocity of money) and its elasticity with respect to the nominal interest rate. To compute these moments, we use the NewM1 monetary aggregate following Lucas and Nicolini (2015). This measure adds to the standard M1 aggregate, published by the Federal Reserve Board, the total amount of Money Market Deposit Accounts held at commercial banks in the US starting from 1982. As discussed by Lucas and Nicolini (2015), money demand measures based on this aggregate perform much better than the ones based on the conventional M1 aggregate. This can clearly be seen in figure 2, where the relationship between money demand and nominal interest rates is much more stable when using the NewM1 aggregate (right panel). In addition, we extend our NewM1 series back in time to January 1948 with the pre-1959 M1 series produced by Rasche (1987). The latter is consistent with the Board's post-1959 methodology.[8]

Regarding the opportunity cost of holding liquidity, we follow Lagos and Wright (2005) in using the monthly Moody's composite yield on Aaa-rated long-term US corporate bonds. Because this measure is non-stationary, we use the HP filter to decompose it into a trend component $\bar{\iota}_t$ and cycle component $\hat{\iota}_t$, i.e.

$$\iota_t = \bar{\iota}_t + \hat{\iota}_t. \tag{58}$$

The cyclical component is modeled as a stationary AR(1) process

$$\hat{\iota}_t = \rho_{\hat{\iota}}\hat{\iota}_{t-1} + \sigma_{\hat{\iota}}\varepsilon_{\hat{\iota},t}, \tag{59}$$

where $\varepsilon_{\hat{\iota}} \sim \mathcal{N}(0,1)$. Its parameters are estimated, at a monthly frequency, to be $\rho_{\hat{\iota}} = 0.939$ and $\sigma_{\hat{\iota}} = 0.0001$. The non-stationary trend component is modeled as a very persistent Markov chain with 5 states. The state values and the estimated transition probabilities are presented in Appendix B.[9]

Following Aruoba et al. (2011), the bargaining power of buyers φ is calibrated internally such that the average markup (i.e. price-to-marginal cost ratio) in DM transactions matches the average markup in the data. De Loecker et al. (2020) use financial statements of all publicly traded firms covering all sectors of the US economy over the period 1955-2016 to estimate an average net markup of 36%. We add their measure to the list of targeted moments. The DM matching efficiency parameter ζ is added to the internal calibration to match a monthly interest rate elasticity of unemployment of 0.297 in the data. Intuitively, this parameter is important for the slope of the inflation-unemployment relationship because of feedback effects between goods and labor markets. For example, an increase in inflation lowers money demand, which, in equilibrium, lowers firm profits and raises unemployment; however, this rise in unemployment lowers the probability of finding a seller in the DM and thus further lowers money demand, and the extent to which it does so depends on goods market frictions.

SMM calibration procedure. The above discussion leaves us with the set of 10 parameters $\{\kappa, b, \chi, \xi, \rho_y, \sigma_y, A, \gamma, \zeta, \varphi\}$ that we calibrate internally following a Simulated Method of Moments (SMM) procedure. The model is solved using a nonlinear global solution method that preserves its strong nonlinearities (Petrosky-Nadeau and Zhang, 2017). Computational details are available

[8]The pre-1959 M1 data series is available from the website of the Federal Reserve Bank of St. Louis (https://research.stlouisfed.org/aggreg/).

[9]Most of the results in our quantitative analysis below have to do with the effect of trend inflation: for example, comparing unemployment volatility in different $\bar{\iota}_t$ regimes. It is nonetheless important to include the cyclical component to generate sufficient variation in nominal interest rates in model simulations; this is essential both when calibrating the model and when comparing its predictions to cross-country regressions.

Table 3: SMM calibrated parameters

Parameter	Description	Value	Moment	Frequency	Data	Model
κ	Vacancy cost	1.471	Average θ	Monthly	0.634	0.634
b	Flow value of unemployment	0.990	Unemployment volatility	Quarterly	0.138	0.138
χ	Parameter of the LM matching fun.	1.269	Average JFP	Monthly	0.430	0.430
ξ	Worker bargaining weight	0.035	Elast. of wage to labor prod.	Quarterly	0.470	0.470
ρ_y	Persistence parameter of y_t process	0.967	Autocorr. of labor productivity	Quarterly	0.758	0.760
σ_y	Volatility parameter of y_t process	0.007	SD of labor productivity	Quarterly	0.013	0.013
A	Level parameter of DM utility	1.421	Average money demand	Quarterly	25.73%	25.72%
γ	Curvature parameter of DM utility	0.217	Elast. of money demand to ι	Quarterly	-0.594	-0.594
ζ	Parameter of the DM matching fun.	0.204	Elast. of u to ι	Monthly	0.297	0.297
φ	Buyer bargaining weight	0.320	Average price markup	Monthly	36.00%	36.00%

Table 4: Labor market statistics

		u	v	θ	$\mathcal{O}$
Quarterly US data, 1948-2019					
Standard deviation		0.138	0.137	0.257	0.013
Autocorrelation		0.895	0.902	0.903	0.758
Correlation matrix	u	1	-0.900	-0.950	-0.231
	v	-	1	0.982	0.363
	θ	-	-	1	0.296
	$\mathcal{O}$	-	-	-	1
Model simulations					
Standard deviation		0.137	0.627	0.740	0.013
Autocorrelation		0.843	0.431	0.636	0.760
Correlation matrix	u	1	-0.559	-0.792	-0.851
	v	-	1	0.903	0.643
	θ	-	-	1	0.758
	$\mathcal{O}$	-	-	-	1

Notes: All variables are reported in logs as deviations from an HP trend with $\lambda = 1600$. *Model-based statistics are computed using 10000 simulations of 1000 months each. For each simulation, we burn the first 136 periods to match the length of the data series. The simulated series are then averaged quarterly. The reported statistics are averages over all simulations.*

in Appendix B.[10] Table 3 summarizes the results of the SMM calibration procedure. In particular, we match all the targeted moments.[11] Table 4 compares some US labor market statistics to those from the calibrated model. The model does a good job in capturing some business cycle properties of the US labor market.

[10]As in Berentsen et al. (2011), the model can exhibit multiple steady state equilibria. We focus our analysis on the high employment equilibrium. We show in Appendix C that it is locally stable. We also plot the ergodic distributions of the endogenous variables to verify that the calibrated economy stays around the high employment equilibrium throughout the simulations.

[11]Our calibrated value of b is is 0.990, which corresponds to a ratio of the flow value of unemployment to average labor productivity of 0.913 – slightly below, e.g. Hagedorn and Manovskii (2008). Note that the value normalized to one is $\overline{y}$, which is less than average output per worker in the model, since the latter includes DM trade.

Discussion. The primary aim of the quantitative analysis below is to examine the state-dependent and nonlinear effects of trend inflation, rather than its average long-run effect on unemployment, which has been discussed at length by e.g. Berentsen et al. (2011). As such, our calibration targets the slope of the long-run relationship between nominal interest rates and unemployment. The model will instead be validated based on its ability to match the state-dependence of this relationship, as evidenced in our empirical analysis.[12]

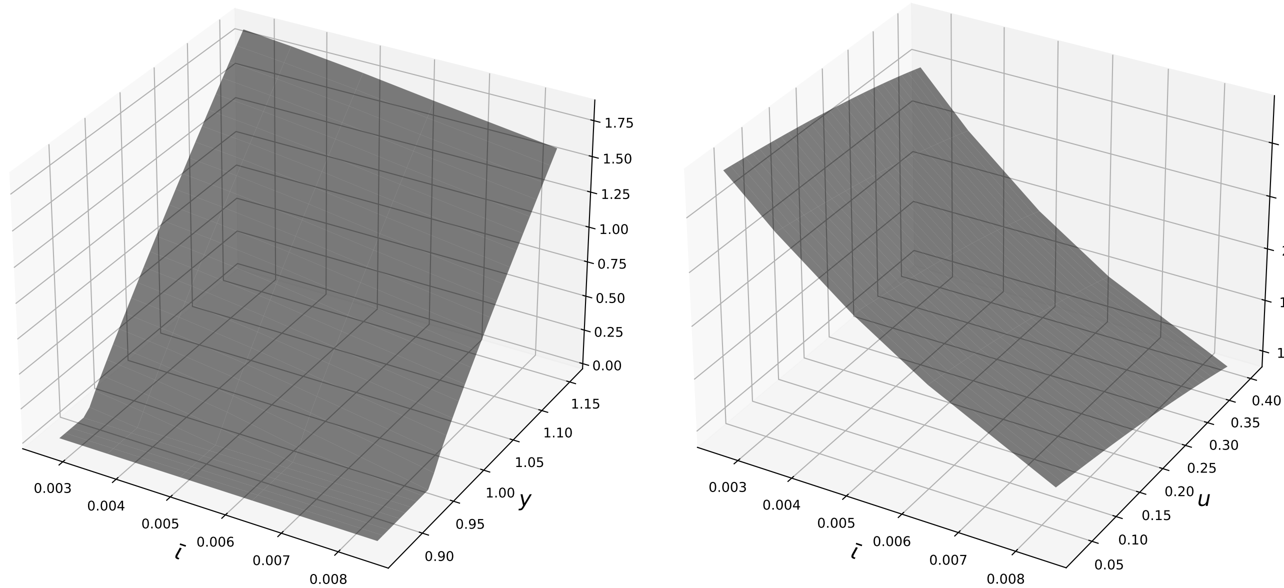
8 Quantitative results

We now show that the quantitative behavior of the model exhibits the mechanisms described theoretically above. We also confirm that the model can generate the correlation patterns we illustrated in the cross-country data. Finally, we use the model to compute the welfare cost of inflation and conduct counterfactual experiments.

Policy functions. The two panels of figure 3 depict the policy functions of the calibrated model, namely LM tightness $\theta(\Omega)$ and DM consumption $x(\Omega)$. The left panel shows θ as a function of trend interest rate $\bar{\iota}$ and CM productivity y. As expected, $\theta(\Omega)$ is decreasing in the interest rate and increasing in CM productivity. The right panel shows the optimal DM consumption $x(\Omega)$ as a function of interest rate $\bar{\iota}$ and unemployment $u = 1 - n$, for a fixed level of productivity. The optimal DM consumption is decreasing in both $\bar{\iota}$ and u. The latter affects x through the DM matching probability since higher unemployment means a lower probability of finding a seller, which incentivizes buyers to carry fewer real balances.

Steady state elasticities. Using the calibrated model we can verify our claims in the theory section regarding the elasticities of θ with respect to productivity and the nominal interest rate. The left panel of figure 4 depicts the numerical steady state elasticity of θ with respect to y in the calibrated model. This elasticity is positive, decreasing in y and increasing in $\bar{\iota}$. It reaches its maximum when productivity is lowest and trend inflation is highest. As we have seen in the left panel of figure 3, this region of the state-space is where θ is at its lowest. This mechanism can also be clearly seen in right panel of figure 4, which depicts $\varepsilon_{\theta,\iota}$, the steady state semi-elasticity of θ with respect to ι. As expected, this elasticity is negative and decreasing in $\bar{\iota}$. In the neighborhood of the Friedman rule, $\varepsilon_{\theta,\iota}$ is unaffected by y, to first order. However, as we deviate from the Friedman rule, it becomes more and more strongly decreasing in y. Labor market tightness θ, and hence unemployment, is most sensitive to changes in nominal interest rates when productivity

[12] Similarly, our calibration targets the level of unemployment volatility, and thus we do not view our analysis here as offering a novel explanation of the unemployment volatility "puzzle." How this volatility varies with the level of inflation, however, is a non-targeted prediction.



(a) $\theta(\Omega)$ in $(\bar{\iota}, y)$ space

(b) $x(\Omega)$ in $(\bar{\iota}, u)$ space $(u = 1 - n)$

Figure 3: Policy functions of the calibrated model.

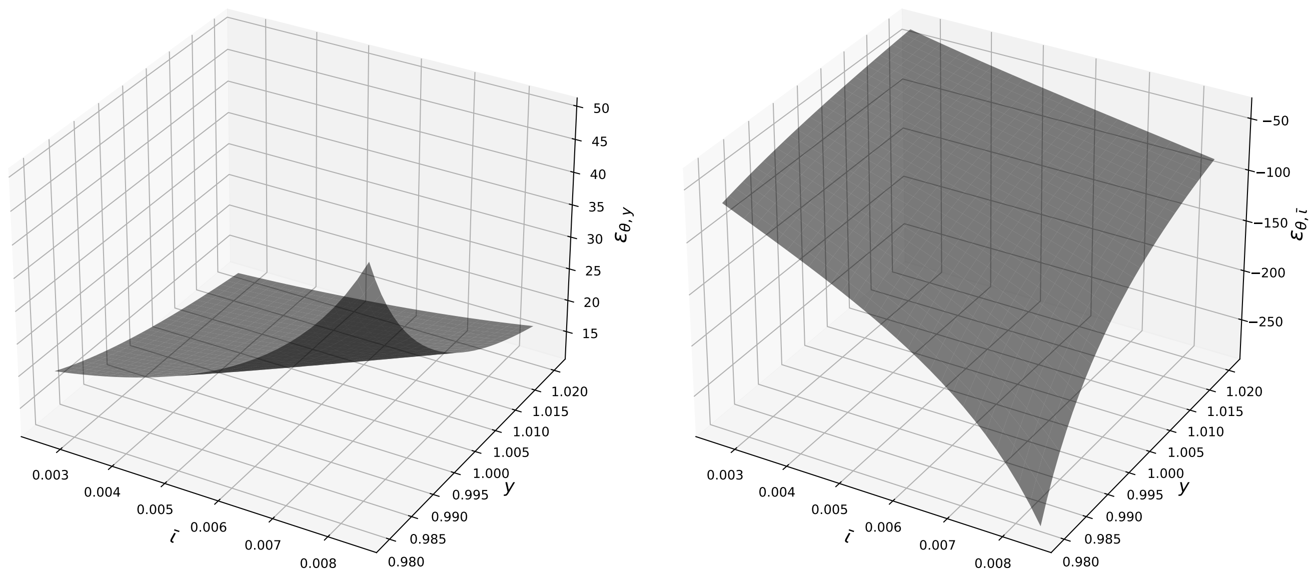
is at its lowest. This suggests, consistent with our findings from the quantile regressions, that unemployment is most sensitive to nominal interest rates at higher quantiles of unemployment, which occur on average at low levels of labor productivity. We confirm this next.

8.1 Nonlinear inflation-unemployment correlations

In order to assess the ability of the model to quantitatively replicate the stylized facts discussed in the empirical section, we simulate the model under both productivity and nominal interest rate shocks.[13] Using the simulated data, we regress unemployment volatility on the trend nominal interest rate. Table 5 presents the results. In particular, a 1pp increase in $\bar{\iota}$ leads to an increase of 0.013 in unemployment volatility. This is very close to the value of 0.01 obtained from the fixed effects panel regression in table 2 using OECD data.

Next, we turn to the state-dependent inflation-unemployment relationship in the calibrated model. Figure 5 shows the coefficients obtained by running OLS and quantile regressions of HP-filtered trend unemployment on trend interest rates using the simulated quarterly data. The slope of the OLS regression stands at 0.43. This is close to the value of 0.35 that we obtained using a pooled OLS regression on OECD data. The coefficients of the quantile regressions computed

[13]We run 1,000 simulations of the model each extending for 1,000 months and drop the first 136 months to match the length of the US data series used in the calibration. We then aggregate the simulated data by taking quarterly averages.



(a) Elasticity of θ wrt. y, $\varepsilon_{\theta,y}$

(b) Semi-elasticity of θ wrt. ι, $\varepsilon_{\theta,\iota}$

Figure 4: Steady state elasticities of θ in the calibrated model.

using the simulated data exhibit a very similar shape to the empirical ones in figure 1. As the level of trend unemployment increases, the effect of an increase in trend interest rate is strengthened. At the 5th percentile of the distribution of unemployment, a 1pp increase in $\bar{\iota}$ leads to a 0.10pp increase in $\bar{u}$. In contrast, at the 95th percentile the same increase in $\bar{\iota}$ leads to a 1.08pp increase in $\bar{u}$ – about a tenfold increase in the effect. In the model, this nonlinear effect is caused by the higher elasticity of job creation when the surplus of the firm-worker match is smaller. The latter occurs in states where unemployment is high, either due to low productivity or high inflation or both.

8.2 Generalized impulse response functions

To further examine the dynamics of the model, and in particular the state-dependent reaction of unemployment and other endogenous variables to shocks, we follow Gallant et al. (1993) and Koop et al. (1996) in computing the generalized, nonlinear, impulse response function

$$GIRF_Y(k, \varepsilon_t, \Omega_t) = \mathbb{E}[Y_{t+k}|\varepsilon_t, \Omega_t = \omega_t] - \mathbb{E}[Y_{t+k}|\Omega_t = \omega_t], \tag{60}$$

where $\Omega_t = \omega_t$ is the state of the economy at the beginning of period t and ε_t is an innovation to the exogenous variable at time t. This function measures the update in the conditional expectation of the variable Y_{t+k} implied by a shock ε_t given the state of the economy $\Omega_t = \omega_t$. The shock ε_t could refer to either the productivity process y_t or the cyclical interest rate process $\hat{\iota}_t$. The GIRF in

Table 5: Regression of unemployment volatility on $\bar{\iota}$ using simulated data

	log unemployment volatility (5y rol. wind. SD)
	(1)
Constant	0.031^{***}
	(0.000)
Trend long-term rate (HP filter)	0.013^{***}
	(0.000)
Observations	269,000
R^2	0.182
	*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are based on model simulations. Unemployment volatility is measured as the 5-year rolling window standard deviation of HP-detrended log unemployment. Long-term nominal interest rate series is filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

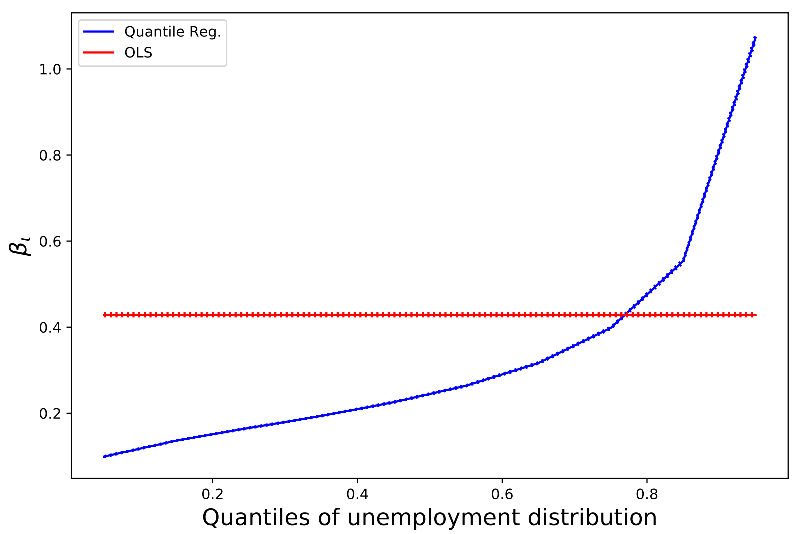
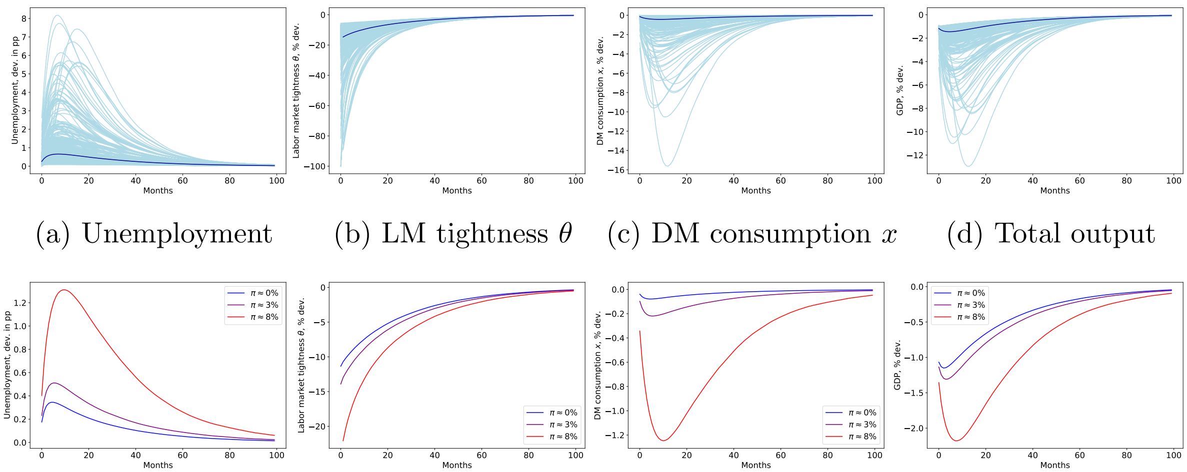


Figure 5: OLS and quantile regressions of trend u on i using simulated data.

equation (60) is a random variable and its shape will in general be a function of $\Omega_t = \{\bar{\iota}_t, \hat{\iota}_t, y_t, u_t\}$, the state of the economy at the moment of the shock. The top row of figure 6 plots the distribution of the GIRFs (light blue) for key model variables in reaction to a shock to the productivity process y_t of size $\varepsilon_t = \sigma_y$. This is done by evaluating the GIRF above at different initial states randomly



(a) Unemployment (b) LM tightness θ (c) DM consumption x (d) Total output

(e) Unemployment (f) LM tightness θ (g) DM consumption x (h) Total output

Figure 6: Reaction to a negative productivity shock: unconditional and conditional GIRFs

Notes: GIRFs for u, θ, x and total output following a 1 standard deviation negative productivity shock. The top row depicts the distribution of GIRFs (light blue) and their mean (dark blue). Each curve represents the GIRF evaluated at a particular initial state drawn randomly from the ergodic distribution of the calibrated model. The bottom row depicts the mean GIRFs conditional on the level of trend inflation being at about 0%, 3% and 8%.

drawn from the ergodic distribution of Ω_t.[14] By averaging across the initial states, one can obtain the mean GIRF given by

$$\mathbb{E}[GIRF_Y(k, \varepsilon_t, \Omega_t)] = \mathbb{E}[Y_{t+k}|\varepsilon_t] - \mathbb{E}[Y_{t+k}], \tag{61}$$

where the expectation operator is taken over the ergodic distribution of the state Ω_t. These are depicted in dark blue in the top row panels of figure 6. Notice that depending on the initial state of the economy the reaction to productivity shocks can be dramatically different from the average. While on average unemployment increases by 0.18pp on impact, that reaction can go up to 0.53pp at the 95th percentile. The reaction of DM consumption is strongly hump-shaped, in particular at higher levels of trend inflation. This is a result of the hump-shaped reaction of unemployment which gradually amplifies goods market matching frictions and reduces money demand.

The bottom row of figure 6 depicts the mean GIRFs conditional on the level of trend inflation. In states where trend inflation is high (in red), the reaction of the economy to shocks is stronger. For example, the average reaction of unemployment on impact is 1.8 times (2.5 times) stronger when trend inflation is about 8% compared to 3% (0%). Labor market tightness reacts on average 1.6 times (2.1 times) stronger under 8% trend inflation compared to 3% (0%).

[14]We take 1,000 random draws from the ergodic distribution of the state vector. For each initial state drawn, we obtain the GIRF by computing the difference between the conditional expectations with and without the shock, each averaged over 10,000 stochastic simulations of the model, of 100 periods' length, starting from that initial state. We use percentage point deviations for unemployment. We use percentage deviations for the other variables.

Table 6: Welfare cost of inflation in the baseline economy

Annual inflation rate	**Implied interest rate**	**Flow welfare level**	**Difference with FR**
-2.75%	0.00%	1.084	-
0.00%	2.82%	1.080	-0.37%
5.00%	7.97%	1.061	-2.13%
10.00%	13.11%	1.035	-4.52%

8.3 Welfare cost of inflation

Our measure of the flow welfare in the model is given by

$$\mathcal{W}(\Omega_t) = \alpha(n_t)[u(x_t) - c(x_t)] + n_t y_t + (1 - n_t)b - \kappa v_t/\beta. \tag{62}$$

Using $v_t = \theta_t(1 - n_{t-1})$ in combination with the law of motion of employment yields:

$$\mathcal{W}(\Omega_t) = \alpha(n_t)[u(x_t) - c(x_t)] + n_t y_t + (1 - n_t)b - \frac{\kappa\theta_t}{\beta}\left(\frac{1 - n_t - \delta}{1 - \delta - f(\theta_t)}\right). \tag{63}$$

By simulating the model, the above equation can be used to calculate average ex-ante welfare in the economy. Because of the highly nonlinear nature of the model, this method provides a better measure of welfare in a stochastic environment compared to the more standard non-stochastic steady state welfare measures.

To measure the welfare cost of inflation, we simulate the model with cyclical interest rate and productivity shocks under various levels of trend inflation. We then calculate the ex-ante welfare for each trend inflation level by averaging across the simulations. Table 6 presents the results at the Friedman rule, 0%, 5% and 10% trend inflation levels. Increasing the trend inflation rate from the Friedman rule to 10% leads to a welfare loss of 4.52%.

Nonlinear cost of inflation. The solid blue line in the left panel of figure 7 plots the welfare level, normalized relative to average welfare at the Friedman rule, as a function of the annual trend inflation rate in the baseline economy. One can clearly see that the slope is decreasing in the trend inflation rate. This means that the effect of a marginal increase in trend inflation becomes stronger as the latter increases. In particular, increasing inflation from 0% to 5% reduces welfare by 1.76pp; increasing it from 5% to 10% leads to an additional 2.39pp welfare loss. This is a 36% stronger effect. In summary, the model predicts that the welfare loss from inflation is significantly nonlinear as well.

The role of aggregate uncertainty. Compared to the previous literature, our model has an additional element that amplifies the welfare cost of inflation: aggregate uncertainty. Its role is

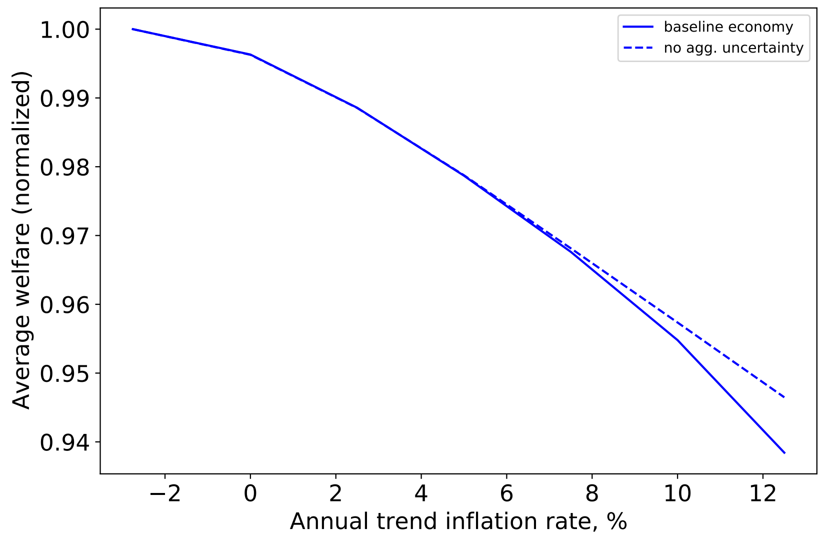


Figure 7: Welfare effects of trend inflation with and without aggregate uncertainty.

related to the observation that higher levels of trend inflation lead to a smaller match surplus, resulting in higher volatility of unemployment following aggregate shocks. Because the reaction of unemployment to shocks is asymmetric, average unemployment under aggregate uncertainty is higher (see e.g. Hairault et al. (2010), Jung and Kuester (2011), and Petrosky-Nadeau and Zhang (2020)). This leads to a lower level of welfare compared to an economy without aggregate shocks. To highlight the effects of this amplification channel on welfare, figure 7 plots in dashed blue the average welfare as a function of trend inflation for an economy where we shut down aggregate uncertainty. The economy without aggregate shocks implies a welfare cost of inflation of 4.26%, as opposed to 4.52% in the baseline economy. That represents an additional 0.26pp or 5.75% of the total welfare cost of inflation in the baseline economy. Furthermore, figure 7 shows that the aggregate uncertainty channel seems to matter only at high levels of inflation (above 7%).

9 Conclusion

In this paper we have argued that there is an important interaction between the inflation tax and business cycles. Such interaction arises naturally in a standard monetary search framework incorporating both labor market frictions and a role for money as a medium of exchange. The analysis also illustrates that such interaction is potentially more important at high levels of inflation. While we deliberately abstract from nominal rigidity and do not explicitly consider stabilization policy, our findings are of potential relevance to recent discussions of raising the inflation target in many countries: an immediate implication of our analysis is that unemployment volatility in response to various shocks is not invariant to this long-run inflation target. More generally, the mechanism we highlight has to do with the interaction between two forces driving unemployment: in our case, inflation lowers the match surplus, which in turn makes unemployment more sensitive to both non-monetary shocks and further increases in inflation. The insight is applicable much more broadly to studying unemployment dynamics in response to multiple shocks.

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Appendix A Tables and figures

Table 7: Regression of $\bar{u}$ on $\bar{\iota}$ (5-year moving averages)

	Trend unemployment (5y moving average)			
	(1)	(2)	(3)	(4)
Constant	5.837***	6.067***	3.005***	2.837***
	(0.597)	(0.385)	(1.302)	(1.029)
Trend long-term rate (5y moving average)	0.366***	0.324***	0.884***	0.915***
	(0.112)	(0.071)	(0.269)	(0.188)
Observations	3,262	3,262	3,262	3,262
R^2	0.083	0.142	0.167	0.200
F-Statistic	295.68***	532.55***	517.21***	744.80***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parentheses. Data are from the OECD. Both unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations by taking 5-year moving averages.

Table 8: Regression of log $\bar{u}$ on trend $\bar{\imath}$ (HP filter)

	Trend log unemployment (HP filter)			
	(1)	(2)	(3)	(4)
Constant	1.707***	1.755***	1.555***	1.705***
	(0.075)	(0.050)	(0.175)	(0.114)
Trend long-term rate (HP filter)	0.039***	0.031***	0.067**	0.039*
	(0.011)	(0.009)	(0.033)	(0.021)
Observations	4,007	4,007	4,007	4,007
R^2	0.072	0.090	0.072	0.024
F-Statistic	312.93***	395.14***	291.58***	92.49***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Both the logarithm of unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

Table 9: Regression of log $\bar{u}$ on $\bar{\imath}$ (5-year moving averages)

	Trend log unemployment (5y moving average)			
	(1)	(2)	(3)	(4)
Constant	1.728***	1.776***	1.493***	1.663***
	(0.082)	(0.045)	(0.156)	(0.098)
Trend long-term rate (5y moving average)	0.041***	0.032***	0.084***	0.053***
	(0.012)	(0.008)	(0.028)	(0.018)
Observations	3,262	3,262	3,262	3,262
R^2	0.075	0.101	0.110	0.052
F-Statistic	263.02***	364.35***	374.11***	164.11***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Both the logarithm of unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations by taking 5-year moving averages.

Table 10: Regression of (HP-detrended) unemployment volatility on $\bar{\iota}$

	Unemployment volatility (HP filter)			
	(1)	(2)	(3)	(4)
Constant	0.390***	0.354***	0.222*	-0.024
	(0.050)	(0.053)	(0.122)	(0.166)
Trend long-term rate (HP filter)	0.046***	0.053***	0.079***	0.128***
	(0.008)	(0.010)	(0.028)	(0.032)
Observations	3,616	3,616	3,616	3,616
R^2	0.077	0.132	0.090	0.135
F-Statistic	301.46***	544.71***	333.41***	519.35***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Unemployment volatility is measured as the 5-year rolling window standard deviation of HP-detrended unemployment. Long-term nominal interest rate series for each country is filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

Table 11: Regression of (5-year moving average detrended) log unemployment volatility on $\bar{\iota}$

	log unemployment volatility (5y moving average)			
	(1)	(2)	(3)	(4)
Constant	0.099***	0.085***	0.064***	0.007
	(0.013)	(0.019)	(0.023)	(0.048)
Trend long-term rate (5y moving average)	0.008***	0.011***	0.015***	0.026***
	(0.003)	(0.004)	(0.005)	(0.009)
Observations	2,882	2,882	2,882	2,882
R^2	0.065	0.113	0.078	0.097
F-Statistic	201.77***	364.07***	224.29***	282.66***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Unemployment volatility is measured as the 5-year rolling window standard deviation of log unemployment detrended using a 5-year moving average. Long-term nominal interest rate series are filtered out of high frequency variations using a 5-year moving average.

Table 12: Regression of (5-year moving average detrended) unemployment volatility on $\bar{\iota}$

	Unemployment volatility (5y moving average)			
	(1)	(2)	(3)	(4)
Constant	0.588***	0.523***	-0.234	-0.740**
	(0.142)	(0.129)	(0.288)	(0.357)
Trend long-term rate (5y moving average)	0.098***	0.110***	0.256***	0.354***
	(0.031)	(0.025)	(0.065)	(0.069)
Observations	2,882	2,882	2,882	2,882
R^2	0.079	0.139	0.196	0.216
F-Statistic	248.16***	460.15***	650.05***	721.46***
Country fixed effects	No	Yes	No	Yes
Time fixed effects	No	No	Yes	Yes
Clustered errors (country level)	Yes	Yes	Yes	Yes

*p<0.1; **p<0.05; ***p<0.01

Notes: Standard errors are in parenthesis. Data are from the OECD. Unemployment volatility is measured as the 5-year rolling window standard deviation of unemployment detrended using a 5-year moving average. Long-term nominal interest rate series are filtered out of high frequency variations using a 5-year moving average.

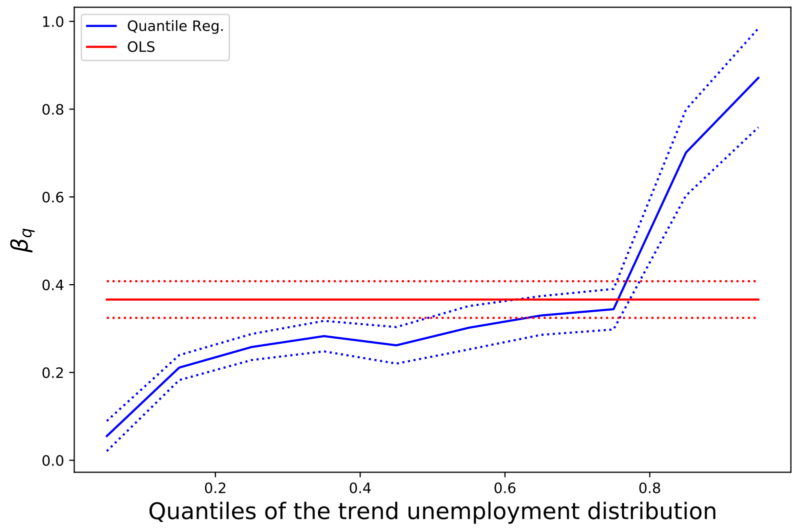

Figure 8: Quantile regression coefficients of $\bar{u}$ on $\bar{\imath}$ for various quantiles of $\bar{u}$ (5-year moving averages)

Notes: The dashed lines represent the 95% confidence intervals. Data are from the OECD. Both unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using a 5-year moving average.

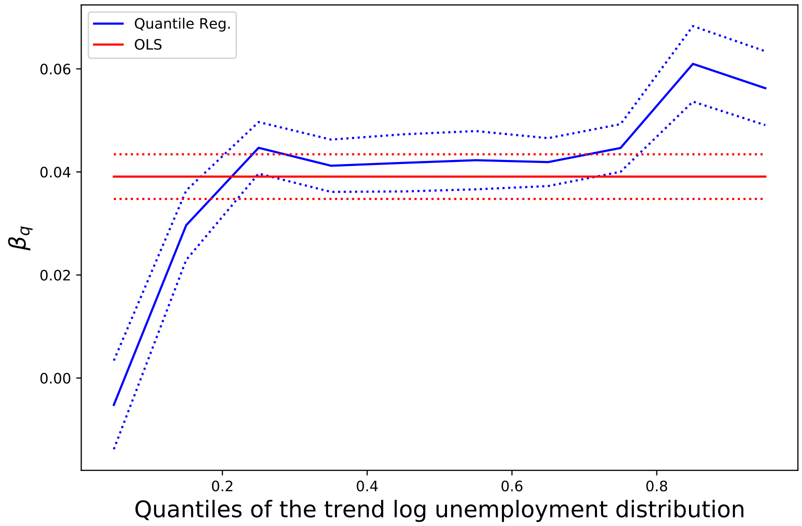

Figure 9: Quantile regression coefficients of trend log u on $\bar{\imath}$ for various quantiles of trend log u (HP filtered)

Notes: The dashed lines represent the 95% confidence intervals. Data are from the OECD. Both log unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using the HP filter with $\lambda = 1600$.

Figure 10: Quantile regression coefficients of trend log u on $\bar{\imath}$ for various quantiles of trend log u (5-year moving averages)

Notes: The dashed lines represent the 95% confidence intervals. Data are from the OECD. Both log unemployment and long-term nominal interest rate series for each country are filtered out of high frequency variations using a 5-year moving average.

Appendix B Computations and calibration

Solution method. The model is solved numerically using value function iteration.[15] We discretize the state space $\Omega_t = \{\bar{\iota}_t, \hat{\iota}_t, y_t, u_{t-1}\}$ as follows: The continuous state stochastic processes for y_t and $\hat{\iota}_t$ are each approximated by a 30-state Markov chain using the Rouwenhorst (1995) procedure, which is shown by Petrosky-Nadeau and Zhang (2017), for the case of productivity shocks, to provide a better approximation when solving the DMP model nonlinearly.[16] The trend component $\bar{\iota}_t$ is modeled as a very persistent Markov chain with 5 states. The state values of the chain are computed by dividing the distribution of $\bar{\iota}_t$ into five quintiles and taking the average value of each quintile. This yields the values: $\{3.24\%, 4.37\%, 6.06\%, 7.74\%, 10.62\%\}$. We then estimate the transition probabilities by maximum likelihood as in Chatterjee and Corbae (2007). The maximum likelihood estimate of p_{jk}, the probability of transitioning from states j to k, is computed as the ratio of the number of times the economy transitions from state j to k to the number of times the economy is in state j. We obtain the following estimated transition probability matrix:

$$\begin{pmatrix} 0.994 & 0.006 & 0 & 0 & 0 \\ 0.006 & 0.988 & 0.006 & 0 & 0 \\ 0 & 0.006 & 0.988 & 0.006 & 0 \\ 0 & 0 & 0.006 & 0.988 & 0.006 \\ 0 & 0 & 0 & 0.006 & 0.994 \end{pmatrix}.$$

The space of the state variable u_{t-1} is discretized using a grid of 30 equidistant points. We linearly interpolate the value function between the grid points of u_t in order to improve accuracy.

Calibration procedure. We calibrate the internal parameters following a Simulated Method of Moments (SMM) procedure.[17] Let Θ be the vector containing the internal parameters, μ the vector of the targeted empirical moments and $\mu_s(\Theta)$ the vector of their model-based counterparts, obtained by simulating the model using a random draw s of productivity and interest rate shocks. The simulated moments are averaged over $S = 1,000$ simulations each of length $T = 1,000$.[18] We burn the first 136 observations to match the length of the empirical data series (864 monthly observations). The SMM procedure consists in solving for the vector $\hat{\Theta}$ that minimizes the distance $G(\Theta) = \mu - \frac{1}{S}\sum_{s=1}^{S} \mu_s(\Theta)$ such that

$$\hat{\Theta} = \arg\min_{\Theta} G(\Theta)^T W^{-1} G(\Theta) \tag{64}$$

[15] Our code is written in Python and uses the Numba library extensively for just-in-time compilation and parallelization (Lam et al., 2015).

[16] We use the Rouwenhorst routine from the QuantEcon Python library (Sargent and Stachurski, 2014).

[17] See for example Ruge-Murcia (2012) and references therein.

[18] To match empirical moments based on quarterly data, we aggregate our monthly simulations quarterly and compute the corresponding model-based moments.

where W is a semi-definite weighting matrix. Since our calibrated model is exactly identified, the choice of the weighting matrix is irrelevant. We use the percent difference to ensure that the distance is unit free and thus avoid unintended weighting.

Appendix C Omitted derivations from section 6

We first derive how x depends on y and ι. Totally differentiating equation (47) with respect to x, y and ι, we find

$$\varphi\left(\frac{\mathrm{c}''(x)}{\mathrm{u}'(x)}-\frac{\mathrm{c}'(x)\mathrm{u}''(x)}{\mathrm{u}'(x)^2}\right)\mathrm{d}x=\left(\frac{\alpha'(n)}{\iota+\alpha(n)}-\frac{\alpha(n)\alpha'(n)}{\left[\iota+\alpha(n)\right]^2}\right)\mathrm{d}n-\frac{\alpha(n)}{\left[\iota+\alpha(n)\right]^2}\mathrm{d}\iota. \tag{65}$$

Rearranging terms yields

$$\varphi\frac{\mathrm{c}'(x)}{\mathrm{u}'(x)}\left(\frac{x\mathrm{c}''(x)}{\mathrm{c}'(x)}-\frac{x\mathrm{u}''(x)}{\mathrm{u}'(x)}\right)\frac{\mathrm{d}x}{x}=\frac{\iota\alpha(n)}{\left[\iota+\alpha(n)\right]^2}\left(\frac{n\alpha'(n)}{\alpha(n)}\frac{\mathrm{d}n}{n}-\frac{\mathrm{d}\iota}{\iota}\right). \tag{66}$$

Using equation (47) to substitute out $\varphi\mathrm{c}'(x)/\mathrm{u}'(x)$ together with $\sigma_{\mathrm{u},x}=-x\mathrm{u}''(x)/\mathrm{u}'(x)$, $\sigma_{\mathrm{c},x}=x\mathrm{c}''(x)/\mathrm{c}'(x)$ and $\epsilon_{\alpha,n}=n\alpha'(n)/\alpha(n)$, yields

$$\frac{\mathrm{d}x}{x}=\frac{1}{\sigma_{\mathrm{u},x}+\sigma_{\mathrm{c},x}}\frac{\iota\alpha(n)}{\iota+\alpha(n)}\frac{1}{\varphi\alpha(n)-(1-\varphi)\iota}\left(\epsilon_{\alpha,n}\frac{\mathrm{d}n}{n}-\frac{\mathrm{d}\iota}{\iota}\right). \tag{67}$$

Note that the term $\varphi\alpha(n)-(1-\varphi)\iota$ is strictly positive whenever we are in the relevant case in which x is strictly positive. Equation (47) namely implies that, keeping everything else constant, x is strictly decreasing in ι with $\lim_{\iota\to\frac{\varphi\alpha(n)}{1-\varphi}}=0$

Next, we totally differentiate $\mathcal{P}$, implicitly defined by equation (46), with respect to x and n:

$$\mathrm{d}\mathcal{P}=\left(\frac{\alpha'(n)}{n}-\frac{\alpha(n)}{n^2}\right)(1-\varphi)\left[\mathrm{u}(x)-\mathrm{c}(x)\right]\mathrm{d}n+\frac{\alpha(n)}{n}(1-\varphi)\left[\mathrm{u}'(x)-\mathrm{c}'(x)\right]\mathrm{d}x. \tag{68}$$

Rearranging terms yields

$$\frac{\mathrm{d}\mathcal{P}}{\alpha(n)(1-\varphi)\left[\mathrm{u}(x)-\mathrm{c}(x)\right]/n}=\left(\frac{n\alpha'(n)}{\alpha(n)}-1\right)\frac{\mathrm{d}n}{n}+\frac{x\left[\mathrm{u}'(x)-\mathrm{c}'(x)\right]}{\mathrm{u}(x)-\mathrm{c}(x)}\frac{\mathrm{d}x}{x} \tag{69}$$

Using that equation (46) implies $\mathcal{P}=\alpha(n)(1-\varphi)\left[\mathrm{u}(x)-\mathrm{c}(x)\right]/n$, that $\epsilon_{\alpha,n}=n\alpha'(n)/n$ and using equation (67) to substitute out $\mathrm{d}x/x$, yields

$$\frac{\mathrm{d}\mathcal{P}}{\mathcal{P}}=\frac{(\epsilon_{\alpha,n}-1)\mathrm{d}n}{n}+\frac{1}{\sigma_{\mathrm{u},x}+\sigma_{\mathrm{c},x}}\frac{\iota\alpha(n)}{\iota+\alpha(n)}\frac{1}{\varphi\alpha(n)-(1-\varphi)\iota}\frac{x\left[\mathrm{u}'(x)-\mathrm{c}'(x)\right]}{\mathrm{u}(x)-\mathrm{c}(x)}\left(\frac{\epsilon_{\alpha,n}\mathrm{d}n}{n}-\frac{\mathrm{d}\iota}{\iota}\right). \tag{70}$$

With $\varepsilon_{\mathcal{P},n}$ defined by equation (49) and $\varepsilon_{\mathcal{P},\iota}$ defined by equation (51), we thus obtain

$$\frac{\mathrm{d}\mathcal{P}}{\mathcal{P}} = \frac{\varepsilon_{\mathcal{P},n}\mathrm{d}n}{n} + \varepsilon_{\mathcal{P},\iota}\mathrm{d}\iota. \tag{71}$$

Note that the sign of $\varepsilon_{\mathcal{P},n}$ is ambiguous while $\varepsilon_{\mathcal{P},\iota} = 0$ if $\iota = 0$ and $\varepsilon_{\mathcal{P},\iota} < 0$ if $\iota > 0$.

We proceed by totally differentiating equation (42) with respect to θ and $\mathcal{O}$:

$$0 = \Upsilon'(\theta)(\mathcal{O} - b)\mathrm{d}\theta + \Upsilon(\theta)\mathrm{d}\mathcal{O}. \tag{72}$$

Rearranging terms and defining $\epsilon_{\Upsilon,\theta} = -\theta\Upsilon'(\theta)/\Upsilon$, which is strictly positive, then yields

$$\frac{\mathrm{d}\theta}{\theta} = \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O} - b}\frac{\mathrm{d}\mathcal{O}}{\mathcal{O}}. \tag{73}$$

Also notice that equation (46) can be totally differentiated to yield

$$\frac{\mathrm{d}\mathcal{O}}{\mathcal{O}} = \frac{y}{\mathcal{O}}\frac{\mathrm{d}y}{y} + \frac{\mathcal{P}}{\mathcal{O}}\frac{\mathrm{d}\mathcal{P}}{\mathcal{P}}. \tag{74}$$

Combining equations (71), (73) and (74) then gives us

$$\frac{\mathrm{d}\theta}{\theta} = \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O} - b}\left[\frac{y}{\mathcal{O}}\frac{\mathrm{d}y}{y} + \frac{\mathcal{P}}{\mathcal{O}}\left(\frac{\varepsilon_{\mathcal{P},n}\mathrm{d}n}{n} + \varepsilon_{\mathcal{P},\iota}\mathrm{d}\iota\right)\right]. \tag{75}$$

Defining $\epsilon_{n,\theta} = \frac{\mathrm{d}n}{\mathrm{d}\theta}\frac{\theta}{n}$ and rearranging terms, we obtain

$$\frac{\mathrm{d}\theta}{\theta} = \left(1 - \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O} - b}\frac{\mathcal{P}}{\mathcal{O}}\varepsilon_{\mathcal{P},n}\varepsilon_{n,\theta}\right)^{-1}\frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O} - b}\left(\frac{y}{\mathcal{O}}\frac{\mathrm{d}y}{y} + \frac{\mathcal{P}}{\mathcal{O}}\varepsilon_{\mathcal{P},\iota}\mathrm{d}\iota\right). \tag{76}$$

It then follows immediately that

$$\frac{\mathrm{d}\theta}{\theta} = \frac{\varepsilon_{\theta,y}\mathrm{d}y}{y} + \varepsilon_{\theta,\iota}\mathrm{d}\iota, \tag{77}$$

with $\varepsilon_{\theta,y}$ given by equation (48) and $\varepsilon_{\theta,\iota}$ given by equation (50).

Finally, totally differentiating equation (43) with respect to θ and n implies

$$\mathrm{d}n = \left(\frac{f'(\theta)}{\delta + f(\theta)} - \frac{f(\theta)f'(\theta)}{[\delta + f(\theta)]^2}\right)\mathrm{d}\theta. \tag{78}$$

Rearranging terms, we obtain

$$\frac{\mathrm{d}n}{\mathrm{d}\theta}\frac{\theta}{n} = \frac{1}{n}\frac{f(\theta)}{\delta + f(\theta)}\frac{\theta f'(\theta)}{f(\theta)}\left(1 - \frac{f(\theta)}{\delta + f(\theta)}\right). \tag{79}$$

Using equation (43) to substitute out the terms $f(\theta)/[\delta + f(\theta)]$ and defining $\epsilon_{f,\theta} = \theta f'(\theta)/f(\theta)$, we obtain

$$\frac{\mathrm{d}n}{\mathrm{d}\theta}\frac{\theta}{n} = \epsilon_{f,\theta}(1-n). \tag{80}$$

This gives us the expression for $\varepsilon_{n,\theta}$.

Finally, we relate the sign of the feedback effect

$$\left(1 - \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\frac{\mathcal{P}}{\mathcal{O}\varepsilon_{\mathcal{P},n}\varepsilon_{\theta,n}}\right)^{-1} \tag{81}$$

in equations (48) and (50) to the dynamic stability of steady-states. For this purpose, we reconsider the free-entry condition in equation (42). Taking into account the equilibrium relationship between $\mathcal{O}$ and n, which operates through the dependency of $\mathcal{P}$ on n, as well as the equilibrium relationship between θ and n, which operates through equation (43), we find that the derivative of the right-hand side of equation (42) with respect to n is given by

$$\Upsilon'(\theta)(\mathcal{O}-b)\frac{\theta}{n}\frac{1}{\varepsilon_{n,\theta}} + \Upsilon(\theta)\mathcal{P}\frac{\varepsilon_{\mathcal{P},n}}{n}, \tag{82}$$

which has the opposite sign of

$$1 - \frac{1}{\epsilon_{\Upsilon,\theta}}\frac{\mathcal{O}}{\mathcal{O}-b}\frac{\mathcal{P}}{\mathcal{O}}\varepsilon_{\mathcal{P},n}\varepsilon_{n,\theta}. \tag{83}$$

Whenever the feedback term is positive, a small increase in employment relative to its steady-state value therefore implies a violation of the steady-state free-entry condition because the value of posting a vacancy becomes too small compared to the cost of posting a vacancy. As a result, market tightness and the job finding rate go down, so that employment gradually converges back to the steady-state. This points towards the local (in)stability of steady-state whenever the feedback effect is positive (resp. negative). In addition, due to the feedback effect, multiple steady-states can exist. The steady-state achieving the highest rate of employment is locally stable, as the right-hand side of the free-entry condition approaches zero when θ approaches infinity.