# Informality, Frictional Markets and Monetary Policy*

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**Abstract**

This paper studies the long run implications of monetary policy on unemployment and output in economies with a large informal sector. I present a monetary model with frictional labor and goods markets and where informality is an equilibrium outcome. Multiple stationary equilibria can exist due to the strategic complementarity between households' demand for money and firms' entry and formalization decisions. I show that unemployment and informality are negatively correlated across these equilibria. In the long run, higher inflation and nominal interest rates lower the demand for money which reduces informality at the cost of higher unemployment. The net effect on the formal sector and tax revenues is ambiguous. I calibrate the model to the Brazilian economy and find that the observed downward trend in inflation and nominal interest rates implies a moderate fall in unemployment and an increase in the size of the informal sector. I simulate the effects of an increase in the long run inflation rate under various government balanced-budget rules. When the additional seigniorage income is used to ease the tax burden on formal firms, higher inflation leads to a reduction in both informality and unemployment.

**Keywords:** informality; money; inflation; unemployment; search and matching.

**JEL classification:** E26, E41, J64, H26, O17.

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# 1 Introduction

Informality is one of the main issues facing developing economies.[1] The informal sector represents on average 32.5% of official GDP with some countries above 50% in Latin America and Africa (Medina and Schneider, 2017). In addition, more than 60% of workers and 80% of firms in the world operate in the informal economy (ILO, 2013). Given the large prevalence of informality, it is not entirely clear how central banks in developing countries should consider it when deciding on monetary policy. This is in particular relevant as many central banks are transitioning to an inflation targeting regime where short run monetary policy interventions are aimed at achieving a given medium to long run inflation target (Ha et al., 2019).

Most of the existing macroeconomic literature has dealt with informality from an optimal taxation perspective where the focus is on the revenue aspects of the inflation tax. Instead, I focus on the effects of changes in the long run inflation rate on the extensive margin, i.e. on firms' entry and the endogenous choice of activity between the formal and informal sectors, and analyze its implications for unemployment, output and tax revenues. For that, I introduce informality into a monetary dynamic general equilibrium model with frictional labor and goods markets. This model combines a two-sector version of the labor search model of Mortensen and Pissarides (1994) with the New Monetarist model of Lagos and Wright (2005). Frictions in the labor market result in equilibrium unemployment and informality while frictions in the goods market provide micro-foundations that make money essential as a medium of exchange. I propose a novel mechanism which links the formalization decision of firms and workers in the labor market to the availability of payment instruments in the frictional goods market. The resulting theoretical framework captures the main stylized facts reported in the literature (Batini et al., 2010; Schneider et al., 2011; La Porta and Shleifer, 2014).

The proposed model features a strategic complementarity between households' demand for money and firms' entry and formalization decisions. I characterize the conditions under which this complementarity results in multiple stationary monetary equilibria. I show that unemployment and informality can be negatively correlated across these equilibria i.e. coordinating on the low unemployment equilibrium instead of the high unemployment equilibrium entails an increase in the relative size of the informal sector.

In this environment, an increase in the long run inflation rate reduces informality at the cost of higher unemployment. Inflation in my model operates through Friedman's real balances channel (Friedman, 1977): higher inflation and nominal interest rates reduce money

[1] Informality is defined here as market-based economic activities which are hidden from the government in order to avoid costly regulation; e.g. taxation, labor protection, etc. This definition excludes home production and criminal activities. See Medina and Schneider (2017) for a related definition.

demand, i.e. the amount of real money balances held by households, which in turn reduces their consumption in the frictional goods market. This has two implications: First, it lowers the expected profits from firms' entry and hence reduces job creation and increases unemployment. Second, it leaves informal firms worse off relative to formal firms as the latter are less affected by liquidity rationing thanks to credit availability in the formal sector. This results in some firms shifting their activity from the informal to the formal sector. Search frictions in the frictional goods market act as an amplification mechanism as the increase in unemployment and the decrease in informality further reduce money demand, consumption, profits and job creation. However, the net effects on the formal sector and government's tax revenues are ambiguous.

I calibrate a stochastic version of the model to the Brazilian economy by matching selected moments from the data. I use the model to answer two questions. First, to what extent can monetary policy account for the low frequency variations in unemployment and informality using the theory presented here? For that, I simulate the counterfactual dynamics implied by the observed path of monetary policy keeping everything else constant. I find that the reduction in the trend of nominal interest rates over the period 1996-2015 contributed to around a fifth of the decrease in trend unemployment and resulted in a small increase in the relative size of the informal sector. Second, what are the long run implications of increasing inflation and the nominal interest rate on output, employment and tax revenues? To answer this question I report the results of increasing inflation from the Friedman rule to an annual rate of 10%: informal employment decreases by 1.7pp at the cost of an increase in unemployment of 1.2pp and a fall in aggregate output of around 3%. I also simulate the implications of changing the inflation target under various balanced-budget rules for the government. In particular, when the additional seigniorage income from higher inflation is used to reduce the tax burden on formal firms and workers, this leads to a reduction in both informality and unemployment.

In my framework, the existence of informality is an equilibrium outcome as formal and informal activities present different implications for workers and firms. Firms choosing to operate formally are subject to taxes but can benefit from government's enforcement of credit contracts which results in higher sales. In contrast, firms choosing to operate informally avoid paying taxes at the risk of being detected and punished by the government. Informal firms are also limited to selling goods against money which lowers their sales when buyers are liquidity constrained. The relative size of the informal sector results endogenously from the joint activity choice of firms and workers faced with these trade-offs. Under mild conditions, I show that high productivity firms operate formally while low productivity firms choose to go underground in line with the empirical evidence reported in the literature.

I also explore some implications of financial development on informality. In particular, a wider availability of credit payments in the formal sector leads to a reduction of the size of the informal sector through two mechanisms: First, it reduces the need for money in the formal sector which in turn reduces money demand and the amount of real balances available for informal transactions. Second, a higher availability of credit makes the formal sector more attractive for firms and hence shifts firm's entry and job creation away from the informal sector.

As more central banks adopt inflation targeting, one important policy implication of this paper is that reducing inflation might have the unintended consequence of increasing the size of the informal sector. This can happen both through a decrease in the opportunity cost of money holdings and an increase in the tax burden on the formal sector as governments compensate for the lost seigniorage income. As a policy recommendation, reducing inflation should be accompanied by measures encouraging financial development in the formal sector to make it more attractive and hence counteract the resurgence of informality. This also points to the importance of understanding and measuring informality for the implementation of monetary policy in developing economies and in particular when choosing the long run inflation target.

The rest of the paper is organized as follows: in the next two sections I present a brief review of the literature and the empirical evidence. The theoretical model is presented in section 4 and the equilibrium solution is characterized in section 5. Section 6 summarizes the main theoretical results of the paper. The calibration procedure and the numerical experiments are presented in section 7. Section 9 concludes.

## 2 Related literature

This paper is related to four strands of the macroeconomic literature: the first one is about informality and optimal taxation, the second one on informality and money, the third one focuses on the long run effects of monetary policy and the fourth one is concerned with the macroeconomic implications of frictions in goods and labor markets.

There is a relatively long tradition of viewing inflation as a tax on informal activities given that informal transactions are more money-intensive compared to formal transactions. Several authors (Canzoneri and Rogers, 1990; Nicolini, 1998; Cavalcanti and Villamil, 2003; Koreshkova, 2006; Aruoba, 2018) show that a deviation from the Friedman rule is optimal in order to smooth taxation between the formal and informal sectors.

Using a two-country version of the cash and credit goods model, Canzoneri and Rogers (1990) argue that optimal taxation might require different inflation rates across countries as a result of differences in the level of informal activity. In the context of a currency union, his argument raises a trade-off between tax efficiency and the potential gains from a common currency. Nicolini (1998) shows that the effects of using inflation to tax the informal economy are quantitatively small even in economies with a large informal sector. In contrast, Cavalcanti and Villamil (2003) show that the optimal inflation rate is substantially above the Friedman rule when the informal sector is large. Similar to Nicolini (1998), both the formal and informal sectors in my model use cash and credit. However, the partitioning between credit and cash goods is endogenized in my model through matching frictions. This creates an additional amplification mechanism which explains in part the larger quantitative effects I find. Koreshkova (2006) presents a cash-in-advance model with costly credit that relates the size of the informal sector to the trade-off between inflation and seigniorage income on the one hand and the tax income on the other hand. She shows that inflation works to smooth the tax burden between the formal and informal parts of the economy. However, she assumes a productivity differential between the formal and informal sectors which makes the size of the informal sector partly exogenous. Aruoba (2018) studies the Ramsey optimal taxation problem in the basic New-Monetarist model with entry (Rocheteau and Wright, 2005) with a focus on state capacity and institutions. He reinterprets the centralized frictionless market as the formal sector and the decentralized frictional market as the tax-evading informal sector. The author introduces an audit technology that allows the government to detect and punish informal transactions. The efficiency of this technology is interpreted as a proxy for the quality of institutions. The benevolent government chooses the least-distorting mix of taxes and inflation as well as the optimal audit effort in order to finance a stream of public expenditures. The model is able to replicate the empirical relationships between institutions on one side and inflation, taxes and tax evasion on the other. Similar to my model, informality is endogenous in Aruoba (2018) and as such the government can use inflation not only as a source of revenue as previously considered in the literature but as a tool to reallocate activity from the informal to the formal sector and hence increase the tax base. Two limits to the interpretation of the Lagos and Wright (2005) model that Aruoba (2018) uses is that the entry of sellers and the use of money in transactions are modeled only for the informal sector. The first assumption cuts any direct links between the formal and the informal sectors.[2] The second assumption overstates the effect of inflation on informality. In comparison, I model explicitly the entry of firms and their decision to operate in the formal or informal sector. I also model the use of money in both formal and informal decentralized

[2] To link the two, Aruoba (2018) assumes a utility function that is non-separable in formal and informal goods. In particular, the two goods are assumed to be substitutes such that an increase in inflation, for example, reduces the consumption of the informal good which automatically increases the consumption of the formal good.

trades and make the formal sector less cash-intensive by introducing partial access to credit. This allows me to study the effects of inflation on the entry of firms and job creation in both the formal and informal sectors.

Fishlow and Friedman (1994) discuss the role tax evasion plays as an intertemporal adjustment margin for households in the absence of consumption smoothing through financial markets. They show that a temporary decrease in current income results in agents evading a larger share of their income. This implies that during recessions tax revenues are expected to fall by more than implied by the reduction in the tax base. I obtain a similar result in my model through a different mechanism. A fall in productivity results in an increase in the size of the informal sector as firms increasingly avoid taxes. This is because lower expected profits imply a lower cost of informality in case of a government audit. Applying the same logic to inflation, Fishlow and Friedman (1994) show that an increase in the inflation tax will result in higher tax evasion as inflation lowers the disposable income of households. This means that a higher level of inflation will be required to finance a deficit when allowing for tax compliance to adjust. I show the existence of an opposing mechanism in economies where tax compliance allows agents to benefit from credit. As inflation increases, using money becomes more costly and agents prefer paying taxes to gain access to credit and avoid inflation.

Using the 2016 demonetization episode in India, Chodorow-Reich et al. (2019) study the interaction between money and informality. The authors focus on the transaction role of money in economic activity using a cash-in-advance model with downward wage rigidity. To test the implications of their theoretical model, they exploit exogenous cross-regional variations in the availability of cash at banks following the demonetization shock. The authors find a substantial short-run impact of the availability of cash on output, employment and credit. They conclude that cash serves an important role in economies with high levels of informality such as India which makes models based on a cashless limit inappropriate for the study of similar economies. In line with their conclusion, I model explicitly the demand for money following the New Monetarist literature (Lagos et al., 2017). Within that literature, Gomis-Porqueras et al. (2014) introduce informality to the basic New-Monetarist model (Lagos and Wright, 2005) by allowing agents to avoid taxes on part of their income through the use of cash in transactions. They derive a model-based measure of informality and produce country-level estimates which tend to be on the lower range of the reduced-form estimates found in the literature. Compared to their work, I add a frictional labor market and focus on the extensive margin in terms of entry and choice of formalization. I also derive a model-based measure of the informal economy which includes labor market aspects. Bittencourt et al. (2014) use instead a monetary overlapping generations model with endogenous tax evasion to study the effect of financial development and inflation on the size of the informal economy. They find that a lower level of financial development provides agents with a higher incentive in participating in tax evasion activities. I also integrate the financial development dimension in the form of a higher availability of credit in the formal sector and obtain a similar result.

The long-run relationship between monetary policy, output and unemployment in advanced economies has been studied extensively from both the theoretical and empirical perspectives. Several authors found compelling empirical evidence against a vertical long run Philips curve in some of these economies (Karanassou et al., 2003; Beyer and Farmer, 2007; Schreiber and Wolters, 2007; Berentsen et al., 2011). In contrast, there are no studies so far on this long run relationship in developing economies. This paper contributes to this literature by studying the long run effects of monetary policy on output and unemployment in an economy with a large informal sector. I find that the long run Philips curve is upward sloping in Brazil in line with the findings of Berentsen et al. (2011) for the US. The optimal long-run inflation rate is discussed extensively in Schmitt-Grohé and Uribe (2010) and references therein. Various results on the optimality of the Friedman rule in the New-Monetarist literature are reviewed in Lagos et al. (2017).

Finally, there is a burgeoning literature that studies the macroeconomic consequences of the interaction between frictions in goods, labor and financial markets in models with explicit micro-foundations both in monetary (Shi, 1998; Berentsen et al., 2011; Gomis-Porqueras et al., 2013; Rocheteau and Rodriguez-Lopez, 2014) and non-monetary economies (Wasmer and Weil, 2004; Bethune et al., 2015; Petrosky-Nadeau and Wasmer, 2013, 2015; Branch et al., 2016; Kaplan and Menzio, 2016). This paper contributes to that literature by studying the interaction between market frictions and informality and how that matters for monetary policy.

# 3 Empirical evidence

Measuring informality faces considerable methodological challenges to the extent that agents operating within the informal economy are relentlessly trying to conceal any traces of their activities. To overcome these challenges, economists resort to indirect measurement methods using electricity consumption, money demand and cash transactions, mismatches in national accounts or household surveys. Below, I highlight some stylized facts related to the macroeconomics of informality as reported by the empirical literature. Batini et al. (2010), Schneider et al. (2011) and La Porta and Shleifer (2014) provide extensive surveys.

**Money and informality:** Schneider (2017) shows cross-country based empirical evidence of a positive correlation between the use of cash in transactions and the size of the informal sector. Campillo and Miron (1997) and Koreshkova (2006) present evidence that inflation is negatively correlated with GDP per capita and that inflation rates are more dispersed among developing countries. There is strong cross-country empirical evidence that inflation is positively correlated with the size of the informal sector (Koreshkova, 2006; Aruoba, 2018). The explanation usually proposed is that weak governments with low tax collection capacity resort to inflation as a means of taxing the cash-intensive and tax-evading informal sector. This is in line with evidence that seigniorage is an important source of government revenues in poor developing countries (Dornbusch and Fischer, 1993).

**Taxation and informality:** Cross-country evidence reported by Johnson et al. (1998), Loayza (1999) and Ihrig and Moe (2004) among others shows a positive correlation between the share of the informal sector in GDP and the corporate tax burden. In addition, Ihrig and Moe (2001) report cross-country evidence suggesting that the size of the informal sector reacts negatively to changes in enforcement and positively to changes in taxation. They show that informality in the manufacturing sector is more responsive to enforcement while it is more responsive to changes in the tax burden in the services sector. The authors explain this difference by the relative difficulty of hiding activity from the government in the manufacturing sector.

**Employment and informality:** Several authors present evidence that high productivity workers sort into the formal sector while low productivity workers tend to be more present in the informal sector (Boeri et al., 2002; Boeri and Garibaldi, 2005; Almeida and Carneiro, 2005). In addition, Boeri and Garibaldi (2005) report evidence based on cross-country and regional data of a positive correlation between unemployment and informal employment. This is in part the result of a higher rate of job separation in the informal sector compared to the formal sector as documented by Bosch and Esteban-Pretel (2012) in the case of Brazil and Maloney (1999) in the case of Mexico. Bosch and Maloney (2008) and Goldberg and Pavcnik (2003) find that most of the reallocation between formal and informal work contracts in Brazil and Colombia occurs between similar types of jobs within narrowly defined industries. This reinforces the thesis of Maloney (1999, 2004) against the segmentation of the labor market along the formal/informal divide. Loayza (1999) and Botero et al. (2004) report evidence of the positive correlation between informality and labor market regulation.

# 4 Model

I consider a setting with discrete time and infinite horizon. In every period, three markets take place sequentially: a decentralized labor market as in Mortensen and Pissarides (1994), called LM, a decentralized goods market following Kiyotaki and Wright (1993) and Lagos and Wright (2005), called DM, and a centralized Walrasian market, called CM. In the CM, trade is a frictionless process and agents take the equilibrium price as given. In the LM and DM, agents must search for matches and bargain pairwise over the terms of trade to share the match surplus.

Two types of agents live infinitely in this model, firms and households, indexed by $f$ and $h$ respectively. Households are of measure 1 and there is an arbitrarily large number of firms, not all of them are active at any point of time.[3] Households enjoy leisure or work, buy and consume goods. Firms maximize profits by posting vacancies, hiring workers to produce goods and selling these goods to the households. Both types of agents discount between periods using the same factor $\beta$. There is no discounting between markets within the period.

An agent (firm or household) can be in one of three states: formal employment, $e$, informal employment, $i$ and unemployment, $u$. I define the value functions at the beginning of the LM, DM and CM as $U$, $V$ and $W$ respectively. These value functions depend on the agent's type $t \in \{f, h\}$, on their current employment status $j \in \{e, i, u\}$, their idiosyncratic productivity $\varepsilon$, and on their net wealth $z$, composed of money holdings $z$ and debt $\ell$.

There are two goods in this economy: the CM good $x$, set to be the numeraire, and the DM good $q$. Both goods are non-storable across periods. There is also an intrinsically useless object $m$ called fiat money that can be stored across periods. The period utility a household obtains from consuming goods and leisure is given by

$$\mathcal{U}_j = v(q) + x + \mathbb{1}_{j \in \{u\}} l$$

for $j \in \{e, i, u\}$ where $\mathbb{1}_{j \in \{u\}}$ is an indicator function that takes value 1 when $j \in \{u\}$ and 0 otherwise. Utility from consuming the CM good is linear while utility from consuming the DM good satisfies the usual assumptions, in particular $v(0) = 0$, $v' > 0, v'' < 0$ and $v'(0) = +\infty$.

**LM** Firms and households meet in the LM and form bilateral work relationships. I assume random matching based on a matching function, $\mathcal{M} = \mathcal{M}(u, v)$, where the number of matches is a function of $u$, the measure of unemployed workers, and $v$, the number of open vacancies.

[3] In what follows, the terms *household*, *worker* and *buyer* are used interchangeably. One could think of a household as being composed of one worker sent to the LM and one buyer sent to the DM and CM.

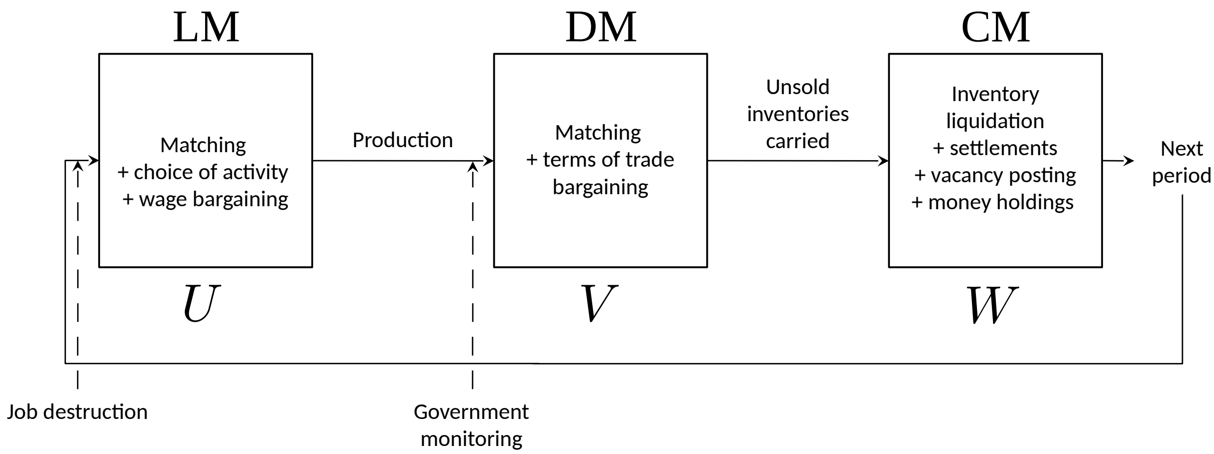



Figure 1: Timeline

The matching function describes the number of new matches resulting from contacts between unemployed workers and firms seeking to fill open vacancies. As is standard in the labor search literature, $\mathcal{M}$ is increasing, concave and homogeneous of degree 1. On the one hand, a firm with a vacancy finds a worker with probability $\alpha_f = \mathcal{M}(u,v)/v = \mathcal{M}(1,\theta)/\theta$ where $\theta = v/u$ is the labor market tightness. On the other hand, an unemployed worker finds a vacancy with probability $\alpha_h = \mathcal{M}(u,v)/u = \mathcal{M}(1,\theta)$. Firms and workers take the aggregate matching probabilities as given.

Workers and firms are ex-ante identical. Unmatched firms can enter the next period's LM by posting a generic job vacancy at the end of the CM which costs $k$ units of the numeraire. Once an unmatched firm meets a worker the idiosyncratic productivity of the match, $\varepsilon$, is revealed. $\varepsilon$ is match specific and reflects the quality of the match. It is drawn from a distribution $F(\varepsilon)$ with continuous and bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$. Given $\varepsilon$, the firm and the worker decide whether to produce and sell formally or informally.[4] I define $n_e$ as the measure of formally employed workers and $n_i$ as the measure of informally employed workers.[5]

Both formal and informal firms produce $\varepsilon y$ units of the CM good that is storable within the period, where $y$ is the aggregate productivity level common to all firms. Output can be transformed into the DM good or sold directly in the CM. Selling quantity $q$ in the DM requires a transformation cost $c(q)$, with $c' > 0$ and $c'' \geq 0$. The remaining inventories

[4]I assume $\underline{\varepsilon}$ is high enough such that the value of a match is always positive. This makes the model more tractable by ruling out post-match separation.

[5]Since each firm employs one worker, these measures correspond as well to the number of formal and informal firms active in the economy.

$\varepsilon y - c(q)$ are sold in the CM. Wages are negotiated in the LM and paid in the following CM. Matched firms and workers are separated with exogenous probability $\delta$.[6]

**DM** Next, firms (sellers) and households (buyers) enter the DM where they can trade $q$ units of the DM good pairwise. Matching in the DM is based on a random matching function, $\mathcal{N} = \mathcal{N}(B, S)$, where $B$ and $S$ are the measures of active buyers and sellers respectively. $\mathcal{N}$ is increasing, concave and homogeneous of degree one. On the one hand, all households take part in the DM as buyers provided they are matched hence $B = 1$. On the other hand, a firm can take part in the DM market as a seller only if it has goods to sell. This is only possible if the firm has managed to recruit a worker in the LM which implies $S = 1 - u$.

A firm meets a buyer (household) with probability $\sigma_f = \mathcal{N}(B, S)/S$. In the same manner, a buyer meets a firm with probability $\sigma_h = \mathcal{N}(B, S)/B$. Since there are formal and informal firms operating in this economy, buyers can be randomly matched with either type of firms. With probability $\frac{n_e}{n_e+n_i}$ the encountered seller is a formal firm and with probability $\frac{n_i}{n_e+n_i}$ an informal firm. To simplify notation, I define the unconditional probability of meeting a formal seller as $\sigma_e = \sigma_h \frac{n_e}{1-u}$ and the unconditional probability of meeting an informal seller $\sigma_i = \sigma_h \frac{n_i}{1-u}$. Notice that the LM and DM are related both through the size and the composition of active firms. Indeed, for a given measure of active firms, an increase in the share of formal (informal) employment in the labor market increases the probability for buyers to meet a formal (informal) firm.

There is no double coincidence of wants in the DM. Commitment is limited and DM meetings are anonymous. These frictions make money essential as a medium of exchange. However, I assume there is a record keeping technology which makes contract enforcement by the government possible.[7] Acquiring the record keeping technology is costless but the enforcement of contracts requires compliance with government regulation. Only firms employing formal labor and paying taxes can benefit from the government's enforcement of their contracts. In particular, formal firms can offer intra-period loans $\ell$ to buyers to be repaid in the subsequent CM.[8] I assume that the record keeping technology is imperfect and available only with exogenous probability $\eta$. $\eta = 1$ means that all formal transactions in the DM are settled in the CM while $\eta = 0$ implies all transactions are immediately settled using money. This allows for the co-existence of credit and money in formal DM transactions.[9] Informal

[6]In appendix A I present an extension of the model with different separation rates for formal and informal jobs.

[7]I assume government punishment in case of default is arbitrarily harsh such that default on contracts is not an option for the parties.

[8]Intra-period loans can be seen as a form of differed payment or supplier credit.

[9]See for example Gu et al. (2016) and Bethune et al. (2019) for other approaches resulting in the coexistence of credit and money.

firms in contrast cannot enforce their contracts and as a consequence resort to immediate settlement using money in the DM.

Informal firms are subject to government monitoring in the DM with probability $\chi \in (0,1)$. In that case, the output of the firm $\varepsilon y$ is seized and destroyed causing the firm to remain inactive until the next period.[10] If the monitored firm is already matched with a buyer, their money holdings are not seized but they remain unmatched until next period's DM. The parameter $\chi$ is a proxy for the government's ability to enforce taxation.

**CM** The CM is a frictionless Walrasian market where the numeraire good is traded. In the CM, firms liquidate what remains of their production, post vacancies, pay wages and distribute their profits as dividends to households. In addition, formal firms pay a lump-sum tax on production $\tau_e$ which reflects the regulatory and fiscal costs imposed by the government on formal activity. Households receive wages and unemployment benefits, buy and consume the CM good, repay their loans and decide how much money to take to the next period.

**Money** Finally, I adopt the following convention regarding money. I define real balances held by an agents as $z = \phi m$ where $\phi = 1/p$ is the endogenous price of money $m$ in terms of the CM good (the inverse of the price level). The aggregate quantity of money that circulates in the economy at any period of time is denoted $M$. New money can be created at no cost by the government. The growth rate of the supply of money is defined as $\gamma = M_{+1}/M$, where the subscript +1 (-1) indicates its value next (last) period. I assume that the government implements changes in $M$ in the CM using lump-sum transfers to households if $\gamma > 1$ and lump-sum taxes on households if $\gamma < 1$.[11] In the main part of the paper I focus on stationary monetary equilibria where the real value of money supply is strictly positive and constant over time such that $\phi M = \phi_{+1} M_{+1} > 0$. This implies that inflation is completely determined by the growth rate of the money supply; i.e. $p_{+1}/p = \phi/\phi_{+1} = \gamma$. Unless otherwise specified, I assume that $\gamma > \beta$ such that the economy is away from the Friedman rule $\gamma = \beta$. I relax the stationarity assumption and consider transitional dynamics in the quantitative sections of the paper.

[10] Assuming that the LM match is destroyed when monitoring is successful is equivalent to having a higher separation rate for informal firms. Appendix A presents a version of the model where that is the case.

[11] I explore the incentive-feasibility of the Friedman rule $\gamma = \beta$ when implemented using taxes on the formal sector in section 6.4. For a related discussion see Andolfatto (2013).

### 4.1 Households

At the beginning of a period, the value for a worker of entering the LM unemployed is

$$U_u^h(0,z) = \alpha_h \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^h(\varepsilon,z), V_i^h(\varepsilon,z)\right\}\ \mathrm{d}F(\varepsilon) + (1-\alpha_h)V_u^h(0,z).$$

With probability $\alpha_h$, an unemployed worker is matched with a firm and the match productivity $\varepsilon$ is then revealed. Depending on $\varepsilon$ the worker and the firm decide jointly on operating formally which yields for the worker the value $V_e^h$ or informally which has value $V_i^h$. With probability $1-\alpha_h$, the unemployed worker is not matched and enters the DM with value $V_u^h$. The value function of an employed worker starting the period with idiosyncratic match productivity $\varepsilon$ and real money balances $z$ is given by

$$U_j^h(\varepsilon,z) = (1-\delta)V_j^h(\varepsilon,z) + \delta V_u^h(0,z), \quad j \in \{e,i\};$$

where $\delta$ is the probability a worker loses his current job and can only search for another one in next period's LM.

The value function of a buyer entering the DM with employment status $j \in \{e,i,u\}$ is

$$\begin{aligned} V_j^h(\varepsilon,z) = \sigma_e &\left\{\eta\left[v(q_c) + W_j^h(\varepsilon, z-d_c)\right] + (1-\eta)\left[v(q) + W_j^h(\varepsilon,z-d)\right]\right\} \\ &+ \sigma_i(1-\chi)\left[v(q) + W_j^h(\varepsilon,z-d)\right] + [1-\sigma_e-\sigma_i(1-\chi)]W_j^h(\varepsilon,z) \end{aligned}$$

which states that she is matched with a formal firms with probability $\sigma_e$; with probability $\eta$, she can use both her own money and a loan from the firm to purchase the quantity $q_c$ for a total price $d_c$ whereas with probability $1-\eta$ she can only use money for the purchase of quantity $q$ at price $d$. With probability $\sigma_i$, she meets an informal firm and if the government fails to monitor the match, which happens with probability $1-\chi$, she can use her money balances to acquire quantity $q$. With complementary probability $1-\sigma_e-\sigma_i(1-\chi)$, the buyer does not find a match and carries her money holdings to the CM.

When a household with employment status $j \in \{e,i,u\}$, productivity $\varepsilon$ and net wealth $a$ enters the CM, they choose consumption $x$ and the amount of real balances carried to the next period $z_{+1} = \phi_{+1}m_{+1} = \phi m_{+1}/\gamma$, by solving

$$W_j^h(\varepsilon,a) = \max_{x, z_{+1}\geq 0}\left\{x + \mathbb{1}_{j\in\{u\}}l + \beta U_j^h(\varepsilon, z_{+1})\right\}$$

subject to

$$x + \gamma z_{+1} = I_j(\varepsilon) + a + \Delta + T$$

where $\Delta$ are the profits distributed by firms, to be defined later, and $T$ is a lump-sum transfer from the government. $I_j$ is the part of a household's income that depends on his employment status with $I_u(0) = b$, $I_e(\varepsilon) = w_e(\varepsilon)$ and $I_i(\varepsilon) = (1-\chi)w_i(\varepsilon)$. Notice that informal workers might not receive a wage if their firm's output is seized by the government which happens with probability $\chi$.[12] Inserting the budget constraint in the objective function, I get

$$W_j^h(\varepsilon, a) = I_j(\varepsilon) + \mathbb{1}_{j\in\{u\}} l + a + \Delta + T + \max_{z_{+1}\geq 0} \left\{-\gamma z_{+1} + \beta U_j^h(\varepsilon, z_{+1})\right\},$$

where $W$ is linear in $A$ and the choice of $z'$ is independent of $A$.[13] Using the linearity of $W_j^h$ in $a$, $V_j^h$ can be written as

$$V_j^h(\varepsilon, z) = \sigma_e \left\{\eta \left[v(q_c) - d_c\right] + (1-\eta)\left[v(q) - d\right]\right\} + \sigma_i(1-\chi)\left[v(q) - d\right] + z + W_j^h(\varepsilon, 0)$$

for $j \in \{u, e, i\}$. Plugging this in the expression for $U_j^h$ and inserting its next period expression into $W_j^h$ I get a recursive formulation for the value function of employed households

$$W_j^h(\varepsilon, a) = I_j(\varepsilon) + a + \Delta + T + Z + \beta\left[(1-\delta)W_j^h(\varepsilon, 0) + \delta W_u^h(0, 0)\right];$$

with $j \in \{e, i\}$ and unemployed households

$$\begin{aligned} W_u^h(0, a) = {} & I_u(0) + l + a + \Delta + T + Z \\ & + \beta\left[\alpha_h \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{W_e^h(\varepsilon, 0), W_i^h(\varepsilon, 0)\right\} \, \mathrm{d}F(\varepsilon) + (1-\alpha_h) W_u^h(0, 0)\right] \end{aligned}$$

where

$$\begin{aligned} Z \equiv \max_{z_{+1}\geq 0} \Big\{ & (\beta - \gamma) z_{+1} + \beta\sigma_e\{\eta[v(q_{c,+1}) - d_{c,+1}] + (1-\eta)[v(q_{+1}) - d_{+1}]\} \\ & + \beta\sigma_i(1-\chi)[v(q_{+1}) - d_{+1}]\Big\} \end{aligned} \tag{1}$$

determines DM consumption given the optimal amount of real money balances households carry to the next period. From $Z$ we can clearly see that a household's money demand is independent of their employment status. In order to solve for the optimal $z_{+1}$, I will have to solve first how the terms of trade in monetary matches $(q, d)$ and credit matches $(q_c, d_c)$ depend on $z$.

[12] I assume informal workers can verify the occurrence of government monitoring. This prevents the case where a firm might untruthfully report a government monitoring to avoid paying the wage to the worker.

[13] This property follows from the assumption of (quasi-)linear preferences (Lagos and Wright, 2005). I also assume transfers are high enough such that $x \geq 0$, $\forall j \in \{e, i, u\}$ given the optimal choice of real balances.

## 4.2 Firms

The value of a firm entering the LM with a vacancy is

$$U_u^f(0) = \alpha_f \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^f(\varepsilon), V_i^f(\varepsilon)\right\} \, \mathrm{d}F(\varepsilon) + (1-\alpha_f)V_u^f(0).$$

where $\alpha_f$ is the probability for a firm of finding a worker. Once the firm and worker are matched, the match-specific productivity level $\varepsilon$ is revealed and the firm and worker decide jointly whether to produce formally with the firm's value as $V_e^f(\varepsilon)$ or informally with the firm's value as $V_i^f(\varepsilon)$. With probability $1-\alpha_f$ the firm is not matched and has the continuation value $V_u^f$. The value of an already matched firm entering the LM with match productivity $\varepsilon$ is

$$U_j^f(\varepsilon) = (1-\delta)V_j^f(\varepsilon) + \delta V_u^f(0), \quad j \in \{e, i\}.$$

At the beginning of the DM, an active firm is matched with probability $\sigma_f$ to a buyer, supplies him with quantity $q$ of the DM good at cost $c(q)$ against payment $d$ expressed in real money balances. Any remaining inventories are sold in the CM. Formal firms can offer the buyer a loan with probability $\eta$ to finance the purchase of quantity $q_c$ at price $d_c$. This leaves us with the following value functions at the beginning of the DM:

$$V_e^f(\varepsilon) = \sigma_f\left[\eta W_e^f(\varepsilon, \varepsilon y - c(q_c), d_c) + (1-\eta)W_e^f(\varepsilon, \varepsilon y - c(q), d)\right] + (1-\sigma_f)W_e^f(\varepsilon, \varepsilon y, 0)$$

for formal firms and

$$V_i^f(\varepsilon) = (1-\chi)\left[\sigma_f W_i^f(\varepsilon, \varepsilon y - c(q), d) + (1-\sigma_f)W_i^f(\varepsilon, \varepsilon y, 0)\right] + \chi\beta U_i^f(\varepsilon)$$

for informal firms where $\chi \in (0,1)$ is the probability that an informal firm is monitored successfully by the government.

The value of entering the CM with productivity $\varepsilon$ carrying inventory $x$ and liquid assets $a$ is

$$W_e^f(\varepsilon, x, a) = x - w_e(\varepsilon) - \tau_e + a + \beta U_e^f(\varepsilon)$$

for formal firms and

$$W_i^f(\varepsilon, x, a) = x - w_i(\varepsilon) + a + \beta U_i^f(\varepsilon)$$

for informal firms. Since holding money is costly and firms have no use for it they carry none of it to the next period. A firm without a worker has nothing to sell and thus does not take part in the DM and CM. However, it can decide to enter the next LM by posting a vacancy at cost $k$ at the end of the CM. This yields the value function

$$V_u^f(0) = W_u^f(0,0,0) = \max\left\{0, -k + \beta U_u^f(0)\right\}.$$

The existence of a congestion externality implies that $\alpha_f$, and hence $U_u^f$, are decreasing in $\theta$. As a consequence, free entry by firms increases $\theta$ to the point where

$$W_u^f(0,0,0) = -k + \beta U_u^f(0) = 0.$$

The above expression can be written as

$$k = \beta\alpha_f \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^f(\varepsilon), V_i^f(\varepsilon)\right\} \, \mathrm{d}F(\varepsilon) \tag{2}$$

where I used $V_u^f(0) = W_u^f(0,0,0) = 0$. The left-hand side of equation (2) represents the cost for a firm of posting a vacancy while the right-hand side represents its discounted expected profits. Using the linearity of $W_e^f$, I rewrite $V_e^f$ as

$$V_e^f(\varepsilon) = R_e(\varepsilon) - w_e(\varepsilon) - \tau_e + \beta U_e^f(\varepsilon).$$

where $R_e(\varepsilon) \equiv \varepsilon y + \sigma_f \left\{\eta\left[d_c - c(q_c)\right] + (1-\eta)\left[d - c(q)\right]\right\}$ is the current period revenue of a formal firm. Inserting the expression for $U_e^f$ and using again $V_u^f(0) = 0$, I get

$$V_e^f(\varepsilon) = R_e(\varepsilon) - w_e(\varepsilon) - \tau_e + \beta(1-\delta)V_e^f(\varepsilon)$$

In the same way, I write the value of an informal firm entering the CM as

$$V_i^f(\varepsilon) = (1-\chi)\left[R_i(\varepsilon) - w_i(\varepsilon)\right] + \beta(1-\delta)V_i^f(\varepsilon)$$

where $R_i(\varepsilon) \equiv \varepsilon y + \sigma_f\left[d - c(q)\right]$ represents the current period revenues for an informal firm if it manages to avoid government monitoring.

## 4.3 Government

Government generates seigniorage income from growing the supply of money at rate $\gamma$ and collects a lump sum tax $\tau_e$ on formal firms' production. In addition, the government collects (distributes) lump sum taxes (transfers) on households $T$. These revenues are used to finance unemployment benefits $b$ and some (wasteful) public spending $G$. The government budget constraint can be written as

$$G + bu = T + \tau_e n_e + (\gamma - 1)\phi M, \tag{3}$$

where the last right hand side term represents seigniorage income. For the moment I assume $T$ adjusts to balance the budget. I will relax this assumption later on.

# 5 Equilibrium

## 5.1 Terms of trade in the DM

The terms of trade in the DM are determined using the proportional bargaining solution due to Kalai (1977).[14] The proportional solution implies that each party receives a constant share of the total surplus $v(q) - c(q)$ proportional to their bargaining power. In a monetary match, the buyer's share is $v(q) - d = \varphi\,[v(q) - c(q)]$ while for the firm it is $d - c(q) = (1-\varphi)\,[v(q) - c(q)]$ where $\varphi \in [0,1]$ is the buyer's bargaining power and $(q, d)$ are the terms of trade, i.e. the quantity traded and the corresponding payment in real balances, that solve

$$\max_{q,d}\; v(q) - d$$

subject to the firm's participation constraint

$$d - c(q) \geq (1-\varphi)\,[v(q) - c(q)] \tag{4}$$

and the liquidity and feasibility constraints

$$d \leq z \tag{5}$$

$$c(q) \leq \varepsilon y. \tag{6}$$

The firm's participation constraint (4) states that they will require at least the share of the total surplus resulting from his bargaining power $1 - \varphi$ to participate in the trade. The real balance constraint (5) states that the buyer cannot spend more money than they are carrying and the feasibility constraint (6) states that the firm cannot sell more goods than it produced in the LM.

Since the buyer gets a positive utility from consuming more he will offer the firm just

[14] I use the proportional bargaining solution instead of the generalized Nash solution in order to avoid the non-monotonicty of the latter. I provide the Nash solution in the appendix. For a discussion of different pricing mechanisms in monetary economies see for example Rocheteau and Wright (2005).

enough to make him participate and hence (4) will always be binding. In addition, I assume $\varepsilon y$ is high enough such that the feasibility constraint (6) is never binding.[15] The problem then simplifies to

$$\max_{q}\ v(q) - c(q)$$

subject to

$$d = \varphi c(q) + (1-\varphi)v(q) \leq z.$$

The resulting proportional bargaining solution is a pair $(q, d)$ that satisfies

$$q = \begin{cases} g^{-1}(z) & \text{if} \quad z < z^* \\ q^* & \text{if} \quad z \geq z^* \end{cases} \quad \text{and} \quad d = \begin{cases} z & \text{if} \quad z < z^* \\ z^* & \text{if} \quad z \geq z^* \end{cases} \tag{7}$$

where $q^*$ is the first best quantity that solves $v'(q^*) = c'(q^*)$, $z^* = g(q^*)$ and

$$g(q) \equiv \varphi c(q) + (1-\varphi)v(q), \tag{8}$$

is a strictly increasing function of $q$. Intuitively, (7) states the following: the buyer acquires the first best quantity $q^*$ and pays the amount $g(q^*)$ as long as $g(q^*) \leq z$, the real amount of money balances they carry. Otherwise, they spend all their money $z$ to acquire $q = g^{-1}(z) < q^*$. Notice that for $q < q^*$, $\partial q/\partial z = \partial g^{-1}(z)/\partial z = 1/g'(q) \geq 0$.

Following the same steps, terms of trade in credit matches solve

$$\max_{q_c, d_c} \quad v(q_c) - d_c$$

subject to the firm's participation constraint

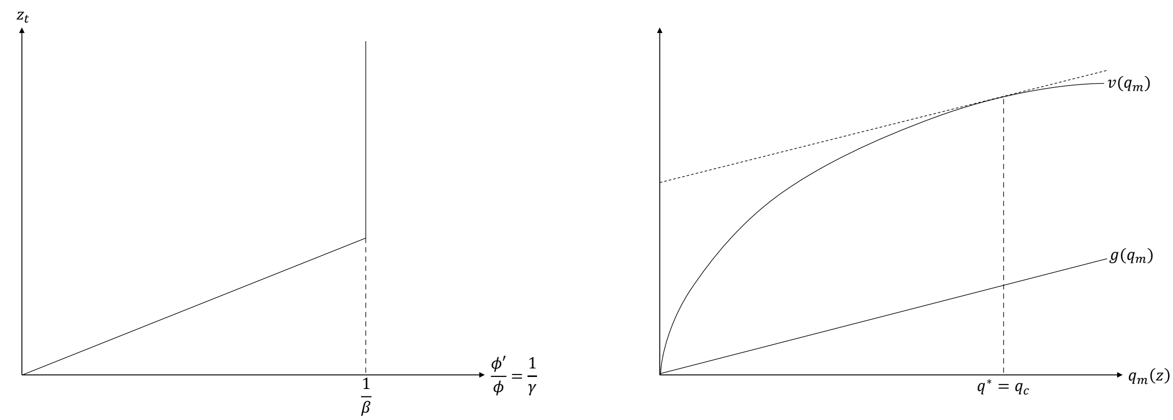
$$d_c - c(q_c) \geq (1-\varphi)[v(q_c) - c(q_c)] \tag{9}$$

and the liquidity and feasibility constraints

$$d_c \leq z + \ell \tag{10}$$
$$c(q_c) \leq \varepsilon y. \tag{11}$$

Notice that the buyer is not constrained by his own real balances and can borrow from the firm in order to finance his purchase. Since buyers don't face a borrowing limit, I can ignore the liquidity constraint (10). Assuming that the feasibility constraint (11) does not bind as

[15] A sufficient condition is $\underline{\varepsilon} y \geq c(q^*)$.




(a) Real balances as a function of money growth (b) DM consumption as function of real balances

Figure 2: Real money balances and terms of trade in monetary DM matches

above, I can rewrite the problem using the binding firm's participation constraint (9) to get

$$\max_{q_c} \; v(q_c) - c(q_c).$$

The resulting proportional bargaining solution is a pair $(q_c, d_c)$ that satisfies

$$q_c = q^* \quad \text{and} \quad d_c = g(q^*) \tag{12}$$

where $g(q)$ is the same as (8) and $q^*$ is the first best quantity defined above. Clearly $q_c$ is independent of the amount of real balances held by the buyer. I assume, without loss of generality, that buyers spend first their money holdings $z$ and then borrow the remaining amount $\ell = g(q^*) - z$ to be repaid in the CM.[16] Since consumption in credit matches $q_c$ always corresponds to $q^*$, I can solve directly for $q_c$ independently of the remaining endogenous variables.

## 5.2 Optimal money balances

Now that I solved for the terms of trade in both monetary and credit matches, I can return to the problem in equation (1) that households face in the CM when choosing the optimal amount of money holdings to carry given the matching probabilities in the next DM. As the

[16] At this point in time, spending money in the DM or instead buying on credit and spending money in the next CM are payoff-equivalent.

pair $(q_c, d_c)$ is independent of $z$ according to (12), problem (1) can be simplified to

$$\max_{z\geq 0} \quad (\beta-\gamma)z + \beta\left\{[\sigma_e(1-\eta)+\sigma_i(1-\chi)]\left[v(q(z))-d(z)\right]\right\}.$$

subject to the bargaining solution given by (7). I start by characterizing the objective function. From (7), we have

$$v'(q)\frac{\partial q(z)}{\partial z} - \frac{\partial d(z)}{\partial z} = \begin{cases} \frac{v'(q(z))}{g'(q(z))} - 1 & \text{for} \quad z < z^* \\ 0 & \text{for} \quad z \geq z^* \end{cases}.$$

where $\frac{v'(q(z))}{g'(q(z))} - 1 > 0$ and $\partial\left[\frac{v'(q(z))}{g'(q(z))}\right]/\partial z < 0$ for all $0 \leq z < z^*$ and $\varphi \in (0,1]$ with $\lim_{z\to 0}\frac{v'(q(z))}{g'(q(z))} - 1 = +\infty$. If $\gamma < \beta$, the objective function is always increasing in $z$ and we have a corner solution $z = +\infty$. This implies that any equilibrium solution must satisfy $\gamma \geq \beta$. It follows that the Friedman rule $\gamma = \beta$ is the minimum money growth rate consistent with an equilibrium.

Assuming $\gamma \geq \beta$, the first order condition yields

$$(\beta-\gamma) + \beta\left\{[\sigma_e(1-\eta)+\sigma_i(1-\chi)]\left[v'(q)\frac{\partial q(z)}{\partial z} - \frac{\partial d(z)}{\partial z}\right]\right\} \leq 0\,, \quad z \geq 0$$

with complementary slackness and we have two cases:

1. If $\gamma = \beta$, any $z \geq z^*$ is a solution and the terms of trade satisfy $q = q^*$ and $d = g(q^*)$.
2. If $\gamma > \beta$, then there is a unique solution $z \in [0, z^*)$ that satisfies

$$(\beta-\gamma) + \beta\left\{[\sigma_e(1-\eta)+\sigma_i(1-\chi)]\left[\frac{v'(q(z))}{g'(q(z))} - 1\right]\right\} = 0$$

with the terms of trade given by $q = g^{-1}(z)$ and $d = z$. The left panel of Figure 2 depicts $z$ as an increasing function of the return on money $1/\gamma$. The right panel depicts consumption in DM monetary matches as a function of $z$.

Notice that in the second case $\gamma > \beta$, there is a one-to-one correspondence between the optimal $q$ and $z$. Given that, I can replace the choice of $z$ with a direct choice of $q$ in the first order condition and rearrange it to get

$$\frac{v'(q)}{g'(q)} = \frac{i}{\sigma_e(1-\eta)+\sigma_i(1-\chi)} + 1 \tag{13}$$

where $i$ is the nominal interest rate defined by the Fisher equation $1 + i = \frac{\gamma}{\beta}$.[17]

[17] The opportunity cost of holding real balances across period $i$ can be interpreted as the nominal rate of

Away from the Friedman rule $q < q^*$ holds and we have the following lemma:

**Lemma 1** *For $\gamma > \beta$ and $\eta > 0$, formal firms have a higher expected DM surplus compared to informal firms such that $R_e(\varepsilon) > R_i(\varepsilon)$. When $\chi > 0$, $R_e(\varepsilon) > (1-\chi)R_i(\varepsilon)$ holds also for $\gamma = \beta$ and $\eta = 0$.*

Intuitively, absent any fiscal or regulatory considerations, enforcement of credit contracts makes formal activity more attractive when immediate settlement using money is relatively costly. In such a case one would expect all firms and workers to operate in the formal sector. The fact that credit is costless here is without loss of generality. Introducing costly credit will just increase the level of $\gamma$ at which this result holds true granted that the cost of credit is not increasing in $\gamma$.

### 5.3 Wage bargaining

The firm and worker choose the type of activity (i.e. formal or informal) that offers the highest present discounted value given their match-specific productivity $\varepsilon$. When $\varepsilon$ is revealed after the firm and worker are matched, the pair have to make two decisions: first, which type of activity to choose and second, what should the wage be. The first decision determines the total match surplus and the second determines the way the total surplus will be split. The two decisions combined determine the surplus of each party. The total match surplus in a job of type $j$ is defined as the sum of net gains for the firm and the worker from being matched

$$S_j(\varepsilon) = V_j^f(\varepsilon) - V_u^f(0) + V_j^h(\varepsilon, 0) - V_u^h(0,0) = V_j^f(\varepsilon) + W_j^h(\varepsilon, 0) - W_u^h(0,0)$$

for $j \in \{e, i\}$ where I used $V_u^f(0) = 0$ and the result that $z$ is independent of $j$.

I use Nash bargaining with termination threat points to decide the wage. This leads to the sharing rule

$$\omega V_j^f(\varepsilon) = (1-\omega)\left[W_j^h(\varepsilon, 0) - W_u^h(0,0)\right] \tag{14}$$

for $j \in \{e, i\}$, where $\omega$ is the bargaining power of the worker and $1-\omega$ that of the firm. Using the model's equations and the bargaining solution (14), I derive the wage equations for formal jobs

$$w_e(\varepsilon) = \omega\left[R_e(\varepsilon) + \theta k - \tau_e\right] + (1-\omega)(b+l) \tag{15}$$

and informal jobs

$$w_i(\varepsilon) = \frac{\omega\left[(1-\chi)R_i(\varepsilon) + \theta k\right] + (1-\omega)(b+l)}{1-\chi}. \tag{16}$$

return on a one-period illiquid bond.

where the wage in both sectors is a convex combination of the firm's surplus generated by the job and the flow value of unemployment foregone by the worker. $\theta k$ represents the economies on search costs that the firm realizes by hiring the worker. Informal wages are adjusted upward to account for the risk of government monitoring.

## 5.4 Formalization decision

I solve now for the optimal choice of activity as a function of the match productivity $\varepsilon$. I use the wage equations (15) and (16) to rewrite the two value functions in steady state as

$$V_e^f(\varepsilon) = (1-\omega)\left[R_e(\varepsilon) - \tau_e - b - l\right] - \omega\theta k + \beta(1-\delta)V_e^f(\varepsilon),$$

and

$$V_i^f(\varepsilon) = (1-\omega)\left[(1-\chi)R_i(\varepsilon) - b - l\right] - \omega\theta k + \beta(1-\delta)V_i^f(\varepsilon)$$

which are both monotonically increasing in $\varepsilon$. This monotonicity implies that the joint formalization decision of firms and workers satisfies a reservation property such that

**Proposition 1** *Under some conditions, there exists a match-specific productivity threshold $\tilde{\varepsilon} \in (\underline{\varepsilon}, \bar{\varepsilon})$ above which matched firms and workers choose to operate formally and below which matched firms and workers choose to operate informally.*

The proof is in Appendix E.1. Proposition 1 is illustrated in Figure 3. In the segment $[\tilde{\varepsilon}, \bar{\varepsilon}]$ of the support of $F(\varepsilon)$, the formal firm's value function $V_e^f(\varepsilon)$ lies above the informal firm's value function $V_i^f(\varepsilon)$ while in the segment $[\underline{\varepsilon}, \tilde{\varepsilon}]$ this order is reversed.[18] Formally, $\tilde{\varepsilon}$ is the productivity level for which the matched firm and worker are indifferent between an informal and a formal work contract such that

$$V_e^f(\tilde{\varepsilon}) = V_i^f(\tilde{\varepsilon}). \tag{17}$$

Solving this expression for $\tilde{\varepsilon}$ (cf. Appendix D.1) yields

$$\tilde{\varepsilon} = \frac{\tau_e - \sigma_f \left\{\eta[g(q_c) - c(q_c)] - (\eta - \chi)[g(q) - c(q)]\right\}}{\chi y} \tag{18}$$

where $\tilde{\varepsilon}$ is a function of $\theta$, $q$, $q_c$ and the model parameters. For $\tilde{\varepsilon} \in [\underline{\varepsilon}, \bar{\varepsilon}]$ to hold, two main

[18] This means that formal jobs have a higher productivity compared to informal jobs in line with the empirical evidence discussed in section 3.

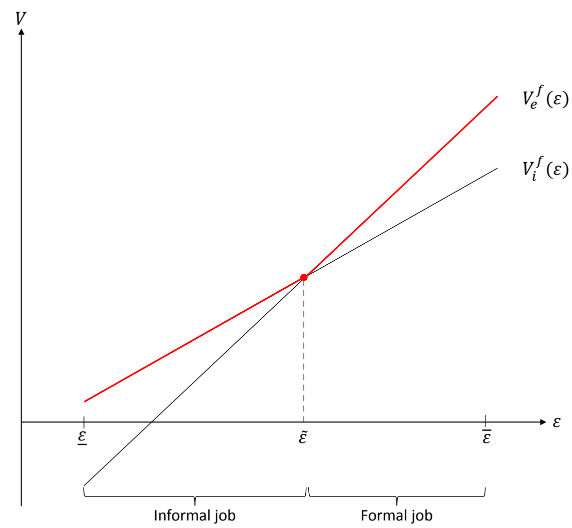



Figure 3: Productivity threshold and hiring decisions.

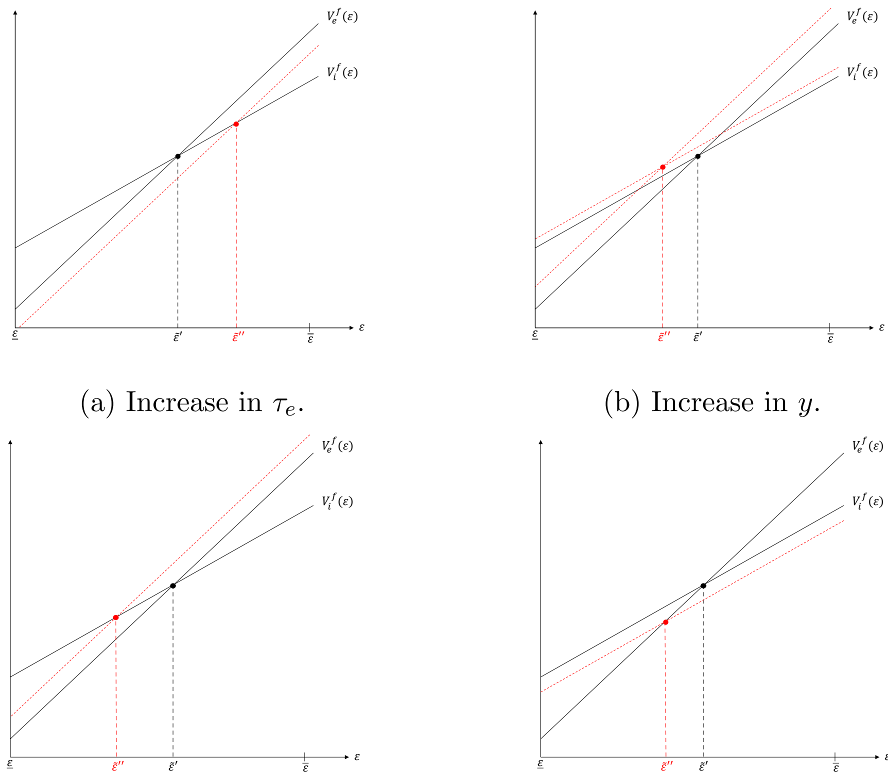
assumptions are needed: First, that government monitoring satisfies $\chi \in (0, 1)$.[19] Second, the level of taxation $\tau_e$ is not too high. Notice that if the level of taxation is too high, all firms and workers choose to be informal ($V_i^f$ lies above $V_e^f$ for all $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$). If the level of taxation is too low such that $\tilde{\varepsilon} \leq \underline{\varepsilon}$, all firms and workers choose to operate in the formal sector ($V_e^f$ lies above $V_i^f$ for all $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$).

Taking the partial derivatives of $\tilde{\varepsilon}$ in equation (18) with respect to the model's parameters returns

$$\frac{\partial \tilde{\varepsilon}}{\partial \tau_e} > 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial \eta} < 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial \chi} < 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial y} < 0.$$

Keeping everything else constant, $\tilde{\varepsilon}$ is increasing in taxes $\tau_e$ and decreasing in availability of credit $\eta$, aggregate productivity $y$ and government monitoring $\chi$. These partial equilibrium results are depicted in figure 4. An increase in taxes, everything else being equal, reduces the profit from a formal job which shifts $V_e^f$ downwards and increases $\tilde{\varepsilon}$. An increase in $\eta$ works in the opposite direction as long as credit is beneficial (i.e. $i > 0$). An increase in $y$ shifts upwards $V_e^f$ by $1/(1 - \beta(1 - \delta))$ and $V_i^f$ by $(1 - \chi)/(1 - \beta(1 - \delta))$. The increase in $V_i^f$ is lower and $\tilde{\varepsilon}$ will decrease. The intuition behind this is straightforward: Informal firms face the risk of losing output due to government monitoring which means an increase in aggregate productivity will translate into a lower expected profits compared to formal firms. As a consequence, firms at the margin will shift their activity from the informal to the formal sector to better benefit from the higher productivity.

[19] In Appendix A, I introduce different job separation rates for formal and informal firms. In that case either $\chi \in (0, 1)$ or $\delta_i > \delta_e$ are necessary for Proposition 1 to hold.



(a) Increase in $\tau_e$.

(b) Increase in $y$.

(c) Increase in $\eta$.

(d) Increase in $\chi$.

Figure 4: Effect of policy parameters on $\tilde{\varepsilon}$.

## 5.5 Stationary monetary equilibrium

As usual in monetary models, there could be multiple stationary equilibria. Since fiat money has no fundamental value and its liquidity value depends on expectations of agents, there is always a non-monetary equilibrium where agents don't value money because they expect others not to do so. Here I focus on stationary monetary equilibria i.e. money is valued and real allocations are constant. In particular, a stationary monetary equilibrium implies a constant real value of the aggregate money supply

$$\phi M = \phi_{+1} M_{+1}$$

and equal in and out-flows in the labor market. The measures of formal employment $n_e$, informal employment $n_i$ and unemployment $u$ evolve according to the following first order difference equations:

$$
\begin{aligned}
u_{+1} &= (1-\alpha_h)u + \delta(1-u); \\
n_{e,+1} &= (1-\delta)n_e + \alpha_h(1-F(\tilde{\varepsilon}))u; \\
n_{i,+1} &= (1-\delta)n_i + \alpha_h F(\tilde{\varepsilon})u,
\end{aligned}
$$

subject to the condition $u + n_e + n_i = 1$ in all periods. It follows that the steady state level of unemployment is given by the standard Beveridge curve (BC)

$$
u = \frac{\delta}{\alpha_h + \delta} \tag{19}
$$

which maps LM tightness $\theta$ through $\alpha_h$ into unemployment, while the composition of employment is given by

$$
n_e = \frac{\alpha_h(1-F(\tilde{\varepsilon}))}{\alpha_h + \delta} \tag{20}
$$

and

$$
n_i = \frac{\alpha_h F(\tilde{\varepsilon})}{\alpha_h + \delta} \tag{21}
$$

both a function of $\tilde{\varepsilon}$ and $\theta$. Using $\tilde{\varepsilon}$ I rewrite equation (2) as the job creation (JC) equation

$$
k = \frac{(1-\omega)\alpha_f \left\{ \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} [R_e(\varepsilon) - \tau_e] \, \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} (1-\chi) R_i(\varepsilon) \, \mathrm{d}F(\varepsilon) - b - l \right\}}{1/\beta - 1 + \delta + \omega\alpha_h} \tag{22}
$$

which determines $\theta$ given firms' expected profits from entry and where $\tilde{\varepsilon}$ satisfies the productivity threshold condition (17). Next, I use the definitions $\sigma_e = \sigma_h \frac{n_e}{n_e + n_i}$ and $\sigma_i = \sigma_h \frac{n_i}{n_e + n_i}$ to rewrite equation (13) as

$$
\frac{v'(q)}{g'(q)} = \frac{i}{\sigma_h \left(1 - \dfrac{\eta + \chi\rho(\tilde{\varepsilon})}{1 + \rho(\tilde{\varepsilon})}\right)} + 1 \tag{23}
$$

where $\rho(\tilde{\varepsilon})$ is the steady state ratio of informal to formal employment given by

$$
\rho(\tilde{\varepsilon}) \equiv \frac{n_i}{n_e} = \frac{F(\tilde{\varepsilon})}{1 - F(\tilde{\varepsilon})}.
$$

In what follows I call equation (23) the money demand (MD) equation because it determines the demand for real balances $z$ and the quantity traded in monetary matches $q$ as a function of the nominal interest rate $i$, the availability of credit $\eta$, government monitoring $\chi$ and the level and composition of employment.[20]

Finally, under the assumption that each household holds a share of a portfolio composed of all active firms in the economy, the equilibrium dividend income $\Delta$ is equal to aggregate profits $\Pi$ given by

$$\Pi = n_e \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon) - \tau_e \,\mathrm{d}F(\varepsilon) + n_i(1-\chi) \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} R_i(\varepsilon) - w_i(\varepsilon) \,\mathrm{d}F(\varepsilon) - u\theta k.$$

where the last term on the right hand side represents the cost of posting vacancies.

**Definition 1** *A stationary monetary equilibrium in this economy consists of (i) a quantity traded in DM credit matches $q_c$, (ii) a quantity traded in DM monetary matches $q$, (iii) a level of LM tightness $\theta$, (iv) a productivity level $\tilde{\varepsilon}$, and (v) a level of unemployment $u$ which together satisfy*

- *The bargaining solution for DM credit matches (12);*
- *The money demand (MD) equation (23);*
- *The job creation (JC) equation (22);*
- *The informality threshold equation (18);*
- *The Beveridge curve (BC) equation (19).*

## 5.6 Equilibrium properties

To describe some properties of the equilibrium solution I start from equation (18) and establish the following lemma:

**Lemma 2** *For $\eta > \chi$, $\tilde{\varepsilon}$ is increasing in $q$ and $\theta$.*

The proof is available in appendix E.2. The intuition is straightforward: When $\eta > \chi$, increasing $q$, keeping $\theta$ constant, increases the profits of informal firms more relative to formal firms and hence shifts firms' entry from the formal to the informal sector. Increasing $\theta$ for a given $q$ increases congestion which reduces matching probability for firms in the DM. Since formal firms make more profits when $\eta > 0$, these firms are hurt more and as such the formal sector becomes less attractive and firms at the margin shift from the formal to the informal sector.

[20] In this model, as in Mortensen and Pissarides (1994), talking about firms or employment is the same since each firm employs a single employee. Because the production function exhibits constant returns to scale the number of workers per firm is irrelevant for the results.

I use equations (18) and (19) to substitute away $\tilde{\varepsilon}$ and $u$ in the JC and MD equations. This allows me to reduce the stationary equilibrium to a system of two equations, JC and MD, in two variables, $\theta$ and $q$. Once the equilibrium values for $\theta$ and $q$ are determined the rest of the endogenous variables can be solved for from the remaining equilibrium conditions listed in Definition 1. Given Lemma 2, the MD equation can be interpreted as an implicit function defining a curve in the $(\theta, q)$ space that maps values of $\theta$ into $q$. The MD curve is described in the following proposition:

**Proposition 2** *For all $i > 0$ and $\eta > \chi > 0$, the MD curve slopes upward in the $(\theta, q)$ space, with $\theta \to +\infty$ implying $q \in (0, q^*)$ and $\theta = 0$ implying $q = 0$. The MD curve shifts down with $i$, $\chi$ and $\eta$ and up with $\varphi$ and $\tau_e$. As $i \to 0$, $q \to q^*$ for all $\theta > 0$.*

A proof is presented in Appendix E.3. In the same way, the JC equation can be interpreted as an implicit function defining a curve that maps values of $q$ into $\theta$ and we have:

**Proposition 3** *The JC curve slopes upward in the $(\theta, q)$ space and passes through $(\bar{\theta}, q^*)$ where $\bar{\theta} \in (0, +\infty)$. Depending on parameters values, it either passes through $(0, \underline{q})$ where $\underline{q} > 0$ or through $(\underline{\theta}, 0)$ where $\underline{\theta} > 0$. It shifts to the left with $\tau_e$, $\chi$, $b$ and $k$ and to the right with $y$.*

A proof is available in Appendix E.4. Figure 5 depicts the MD and JC curves in the $(\theta, q)$ space. The JC curve depicts $\theta$ as increasing in $q$. This is because the more real balances firms expect households to bring to the DM the higher expected profits will be and the more firms will enter, hence a higher $\theta$. The shape of the JC curve reflects the concavity of the matching function. The MD curve depicts $q$ as an increasing function of $\theta$. The transmission from LM to DM works here through the DM matching probability. Increasing $\theta$ increases the chance of meeting a firm in the DM and hence the return on holding money i.e. money can be spent on the DM good with a higher probability. The shape of the DM curve reflects the concavity of the DM matching function.

Depending on where the two curves intersect, there could be one or two monetary equilibria. This multiplicity of equilibria is a result of the strategic complementarity between the entry decision of firms and the demand for money by households. According to Proposition 3, when the JC curve passes through $(0, \underline{q})$, the two curves intersect in two points: a high and a low monetary equilibrium. In the high equilibrium, households expect a higher level of firms entry and hence more frequent trading in the DM. For that reason, households bring a higher amount of money holdings and end up consuming a higher quantity of the DM good. At the same time, firms expect a high level of demand in the DM and hence higher trading surplus which results in higher firms entry and job creation. The intuition is reversed for

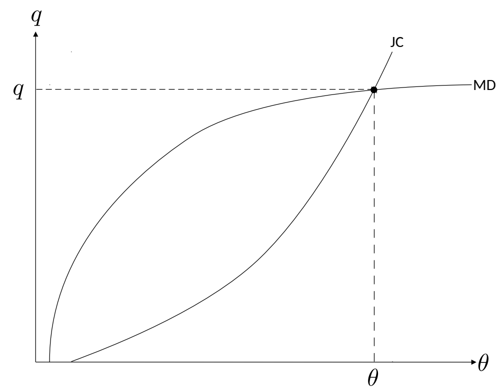



Figure 5: Steady State Equilibria in $(\theta, q)$ space

the low equilibrium. Clearly these two equilibria can be Pareto-ranked with the high equilibrium providing lower unemployment and higher consumption while the low equilibrium exhibiting higher unemployment and lower consumption. The latter can be interpreted as a coordination failure. For parameter values where the JC curve passes through $(\underline{\theta}, 0)$, the two curves will intersect only at the high equilibrium. This is because even when $q = 0$, some firms still enter the LM to earn the expected profit from credit matches and sales in the CM. For parameter values where two stationary monetary equilibria exist I have the following proposition:

**Proposition 4** *When the JC curve passes through* $(0, \underline{q})$*, where* $\underline{q} > 0$*, it intersects twice with the MD curve. In this case two stationary monetary equilibria exist. The high equilibrium corresponds to low* $u$ *and high* $\tilde{\varepsilon}$ *while the low equilibrium corresponds to high* $u$ *and low* $\tilde{\varepsilon}$*.*

A proof is available in Appendix E.5. Proposition 4 implies a negative correlation between unemployment and informality across the two stationary monetary equilibria. The intuition behind it is that informality is increasing in the amount of real balances carried by households. As such, when firms expect households to carry a large amount of real balances not only more firms enter but also more firms choose to operate informally as informal activity becomes more profitable. In the same way, if households expect more firms to enter and operate in the informal sector, they will demand more real balances as they expect more opportunities to trade with money.

### 5.7 Efficiency

I define the constrained efficient allocation as the one that maximizes the utility of households subject to search and informational frictions. If we abstract from government policy considerations, the social planner is indifferent between allocating firms and workers to the formal or informal sectors, as the two are defined only with respect to taxes, government monitoring and credit enforcement, all government policies. In such a case, the social planner chooses the quantity consumed in DM matches $q$ as well as labor market tightness $\theta$ taking as given last period's unemployment by solving[21]

$$\mathcal{W}(u) = \max_{q\geq 0, \theta \geq 0} \left\{ -\frac{\theta u k}{\beta} + u_{+1} l + (1-u_{+1}) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon) + \sigma_h [v(q) - c(q)] + \beta \mathcal{W}(u_{+1}) \right\}$$

subject to

$$u_{+1} = \delta(1-u) + (1-\alpha_h(\theta))u.$$

The first order conditions require that

$$v'(q) - c'(q) \leq 0, \quad q \geq 0$$

$$\frac{-uk}{\beta} + \frac{\partial u_{+1}}{\partial \theta} \left[ l - \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon) + \beta \frac{\partial J(u_{+1})}{\partial u_{+1}} \right] + \frac{\partial \sigma_h}{\partial \theta} [v(q) - c(q)] \leq 0, \quad \theta \geq 0$$

with the corresponding complementary slackness conditions. Taking the envelope condition with respect to $u$ yields

$$\frac{\partial J(u)}{\partial u} = -\frac{\theta k}{\beta} + (1-\alpha_h - \delta) \left\{ l - \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon) + \frac{\partial \sigma_h}{\partial u_{+1}} [v(q) - c(q)] + \beta \frac{\partial J(u_{+1})}{\partial u_{+1}} \right\}.$$

Assuming an interior solution at the steady state equilibrium, I combine the first order condition with respect to $\theta$ and the envelope condition above to get

$$k = \frac{\frac{\partial \alpha_h}{\partial \theta} \left\{ \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon) - \frac{\partial \sigma_h}{\partial u} [v(q) - c(q)] - l \right\}}{1/\beta - 1 + \delta + \alpha_h - \frac{\partial \alpha_h}{\partial \theta} \theta}$$

which, together with the first order condition with respect to $q$, determine the constrained efficient allocation $(\theta^*, q^*)$.

Maintaining the assumption of lump-sum taxes in the CM, I impose that the decentralized

[21] See Gomis-Porqueras et al. (2013) for a similar formulation.

equilibrium solution is equal to the social optimum which yields

$$\frac{\frac{\partial\alpha_h}{\partial\theta}\left\{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\varepsilon y\,dF(\varepsilon)-\frac{\partial\sigma_h}{\partial u}[v(q^*)-c(q^*)]-l\right\}}{1/\beta-1+\delta+\alpha_h-\frac{\partial\alpha_h}{\partial\theta}\theta^*}=$$

$$\frac{(1-\omega)\alpha_f\left\{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}[R_e(\varepsilon)-\tau_e]\ \mathrm{d}F(\varepsilon)+\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}(1-\chi)R_i(\varepsilon)\ \mathrm{d}F(\varepsilon)-b-l\right\}}{1/\beta-1+\delta+\omega\alpha_h}$$

where $\tilde{\varepsilon}$ is given by (18) and

$$v'(q^*)-c'(q^*)=v'(q)-\left\{\frac{i}{\sigma_h\left(1-\dfrac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)}+1\right\}g'(q)$$

where $g(q)$ is given by (8). It is clear that the second equality can be obtained by imposing the Friedman rule $i=0$ which yields $q=q^*$.[22] Using that in the first equality yields

$$\frac{\frac{\partial\alpha_h}{\partial\theta}\left\{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\varepsilon y\,dF(\varepsilon)-\frac{\partial\sigma_h}{\partial u}[v(q^*)-c(q^*)]-l\right\}}{\frac{1}{\beta}-1+\delta+\alpha_h-\frac{\partial\alpha_h}{\partial\theta}\theta^*}=\frac{(1-\omega)\alpha_f}{1/\beta-1+\delta+\omega\alpha_h}$$
$$\left\{\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\varepsilon y\,dF(\varepsilon)+\sigma_f[g(q^*)-c(q^*)]-\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}\tau_e\ \mathrm{d}F(\varepsilon)-\chi\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}\varepsilon y+\sigma_f[g(q^*)-c(q^*)]\ \mathrm{d}F(\varepsilon)-b-l\right\}.$$

First, we have that

$$\frac{\partial\alpha_h}{\partial\theta}=(1-\omega)\alpha_f$$

from the numerator and

$$\alpha_h-\frac{\partial\alpha_h}{\partial\theta}\theta=(1-\omega)\alpha_h$$

from the denominator must both hold. Replacing by $\alpha_h=\mathcal{M}(1,\theta)$ and $\alpha_f=\mathcal{M}(1,\theta)/\theta$, it is easy to see that the two expressions are equivalent and simplify to

$$1-\omega=\frac{\partial\mathcal{M}(1,\theta)}{\partial\theta}\frac{\theta}{\mathcal{M}(1,\theta)}$$

where the right hand side is the elasticity of the matching function to $\theta$. This is the Hosios (1990) condition known in the search literature. Intuitively, given the technology of the LM

[22] I maintain here the assumption that lump-sum taxes in the CM are available.

matching function, decentralizing the social optimum requires that the share of each party in the match surplus is set equal to their marginal contribution to the creation of a match. Second, I need that

$$-\frac{\partial \sigma_h}{\partial u}[v(q^*) - c(q^*)] = \sigma_f(1-\varphi)[v(q^*) - c(q^*)]$$

holds where I replaced $g(q)$ by its expression from (8). This implies that

$$1 - \varphi = \frac{\partial \mathcal{N}(1, 1-u)}{\partial(1-u)} \frac{1-u}{\mathcal{N}(1, 1-u)}$$

where I used $\sigma_h = \mathcal{N}(1, 1-u)$ and $\sigma_f = \mathcal{N}(1, 1-u)/(1-u)$. This is a Hosios condition for the DM similar to the one for the LM.

Finally, we have that

$$\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \tau_e \, \mathrm{d}F(\varepsilon) + \chi \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon y + \sigma_f[g(q^*) - c(q^*)] \, \mathrm{d}F(\varepsilon) + b = 0$$

must hold given the expression for $\tilde{\varepsilon}$ (18). The planner can set $b = 0$ and either set $\chi = 0$ which yields $\tilde{\varepsilon} = \bar{\varepsilon}$, i.e. all firms and workers operate informally, or set $\tau_e = 0$ which yields $\tilde{\varepsilon} = \underline{\varepsilon}$, i.e. all firms and workers operate formally, or both $\chi = \tau_e = 0$ which makes $\tilde{\varepsilon}$ indeterminate. In all three cases $\tau_e n_e = 0$. Indeterminacy when $\tau_e = \chi = 0$ makes sense as in such a laissez-faire economy, the concept of informality is irrelevant. Indeed, the only thing that makes the formal sector attractive from a social welfare point of view is the capacity of the government to enforce credit contracts. However, at the Friedman rule, credit is irrelevant as agents carry the optimal quantity of real balances. I summarize these findings in the following proposition:

**Proposition 5** *Given no constraints on policy, the socially optimal allocation can be achieved by setting $i = 0$, satsifying the Hosios efficiency condition in both LM and DM, setting $b = 0$ and either $\tau_e = 0$, $\chi = 0$ or both.*

Now if for whatever reason $\tau_e > 0$ and $\chi > 0$, the planner can set $b < 0$ to restore efficiency. Intuitively, unemployment can be taxed in order to increase entry and job creation.

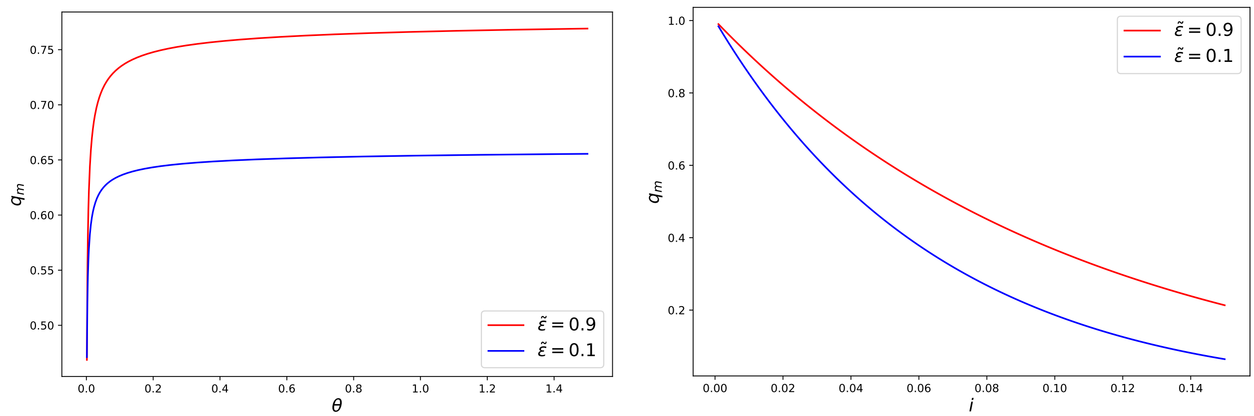



Figure 6: Quantity traded in DM monetary matches and informality.

# 6 Comparative statics and theoretical results

## 6.1 Informality and money demand

The MD equation (23) is an extension of the standard money demand equation present in most monetary models and in particular those with search and matching frictions (Lagos and Wright, 2005; Berentsen et al., 2011) to economies with an informal sector. For parameter values where the informal and formal sectors coexist (i.e. $\tilde{\varepsilon} \in (\underline{\varepsilon}, \bar{\varepsilon})$) and when some or all transactions in the formal sector are settled using credit, it is easy to see from equation (23) that

$$\frac{\partial q}{\partial \tilde{\varepsilon}} > 0$$

when $\eta > \chi > 0$. This means that everything else being equal, a policy that results in an increase in informality will lead to higher demand for money and higher consumption in DM monetary transactions. Intuitively, an increase in the steady state ratio of informal to formal firms in the DM will reduce the frequency of matches where credit is feasible. Buyers will respond by carrying more real balances to the DM. Both panels of figure 6 depict an upward shift in the demand for money when the level of informality increases $\tilde{\varepsilon}$.

## 6.2 Informality and the Beveridge curve

Does the level of informality matter for unemployment? The standard textbook Beveridge curve (BC) describes the long-run steady state unemployment level as a function of firms entry $\theta$ and the exogenous separation rate (Pissarides, 2000). Firms entry affects unemployment through the number of matches created which is an increasing function of $\theta$. The more firms enter the labor market and post vacancies the higher the number of jobs created and

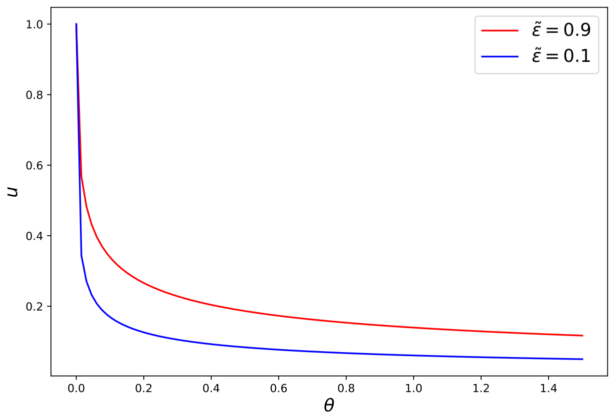



Figure 7: The Beveridge curve for high and low informality.

the lower unemployment. When job separation rates are different for formal and informal jobs, steady state unemployment not only depends on firms entry decision but also on their activity decision. Equation (A.1) depicts the "informality-augmented" BC as a function of both $\theta$ and the ratio of informal to formal employment $\rho(\tilde{\varepsilon})$. In particular, equation (A.1) implies

$$\frac{\partial u}{\partial \tilde{\varepsilon}} > 0$$

for $\delta_i > \delta_e$. This means that a policy that leads to an increase in informality, everything else being equal, will lead to an upward shift in the Beveridge curve as seen in Figure 7. When informal jobs have a higher separation rate; i.e. $\delta_i > \delta_e$, an increase in the ratio of informal to formal jobs $\rho(\tilde{\varepsilon})$, keeping the rest constant, increases the average rate of job destruction in the pool of existing jobs which increases unemployment for all levels of $\theta \in (0, +\infty)$.

## 6.3 Monetary policy

What are the long run implications of an increase in the inflation rate $\gamma$ and the nominal interest rate $i$ on the extensive margin i.e. firms' entry and formalization decisions? The following proposition provides an answer:

**Proposition 6** *Under certain conditions on parameters, an increase in $\gamma$ and $i$ leads to a decrease in $\theta$ and $\tilde{\varepsilon}$. This translates into higher $u$ and lower $n_i$. The effect on $n_e$ is ambiguous.*

The proof is available in Appendix E.6. From Proposition 2 we know that an increase in $i$ reduces the quantity traded in monetary matches $q$.[23] We also know from Proposition 3

[23] The transmission from $i$ to $q$ depends as well on the trading protocol used to share the DM surplus (Rocheteau and Wright, 2005; Aruoba et al., 2007; Craig and Rocheteau, 2008).

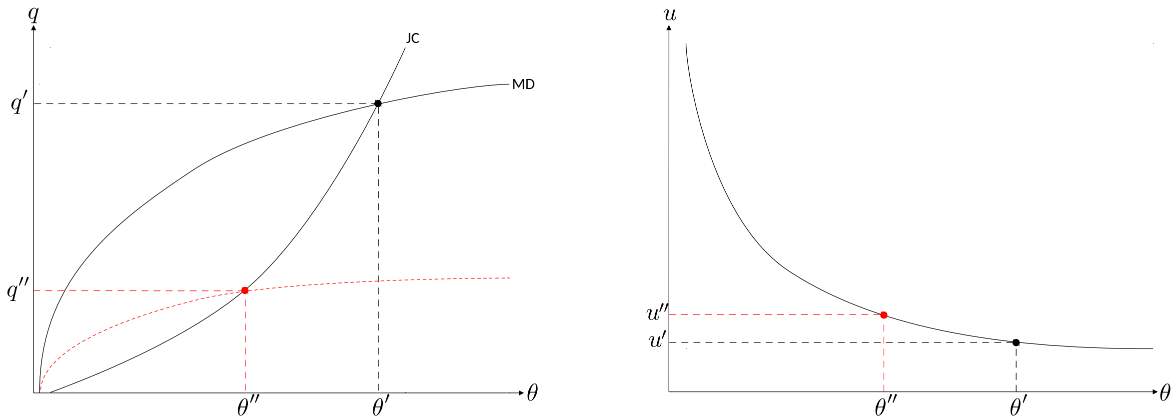



Figure 8: Effect of an increase in $i$ on unemployment without informality.

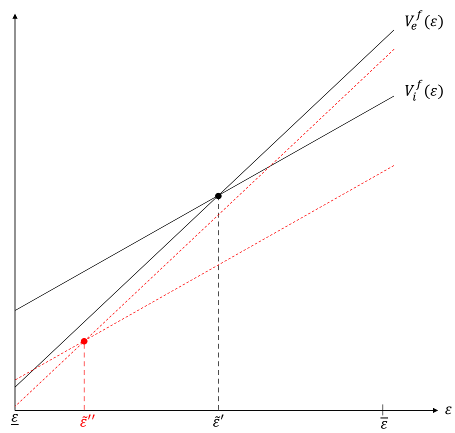



Figure 9: Effect of an increase in $i$ on $\tilde{\varepsilon}$

that a decrease in $q$ leads to reduced profits for firms and hence lower entry and market tightness $\theta$. Less entry and jobs creation translates into higher unemployment. Because the matching probability in the DM depends on the number of active firms, less entry of firms further reduces the demand for money by households which feeds into lower profits in the DM and amplifies the effects of the initial increase in $i$ on unemployment and output.

The first panel in Figure 8 depicts this result in terms of the MD and JC curves. An increase in $i$ translates into lower $q$ for all levels of $\theta$ resulting into a downward shift of the MD curve. Since $i$ does not enter directly equation (22), the JC curve does not move and the two curves will cross at a lower point $(\theta'', q'')$. A lower $\theta$, everything else being equal, results in a higher level of unemployment through the Beveridge curve (19) as illustrated in the second panel of Figure 8.

In addition to the effect of inflation on firms' entry and unemployment, Proposition 6 states a second effect related to the composition of the economy. A higher inflation tax reduces more the expected sales and profits of informal firms compared to formal firms as the latter benefit from the availability of credit in the formal sector. This is illustrated in Figure 9 where an increase in $i$ translates into $V_i^f(\varepsilon)$ shifting downward more than $V_e^f(\varepsilon)$. Conditional on entry, this pushes $\tilde{\varepsilon}$ to the left of the support of the productivity distribution. As the threshold changes, more firms and workers reallocate their activity from the informal to the formal sector which lowers $n_i$. The net effect on $n_e$ is ambiguous. As seen from the expression

$$\frac{dn_e}{di} = \underbrace{\frac{[1-F(\tilde{\varepsilon})]\delta}{[\alpha_h+\delta]^2}\frac{d\alpha_h}{d\theta}\frac{d\theta}{di}}_{\text{Entry margin}} - \underbrace{\frac{\alpha_h}{\alpha_h+\delta}\frac{dF(\tilde{\varepsilon})}{d\tilde{\varepsilon}}\frac{d\tilde{\varepsilon}}{di}}_{\text{Formalization margin}},$$

the outcome depends on whether the formalization margin dominates the entry margin or the opposite.

When the separation rate is higher in the informal sector, $\delta_i > \delta_e$, another general equilibrium effect takes place. We know from Proposition 6 that higher $\gamma$ and $i$ shift $\tilde{\varepsilon}$ to the left. The Beveridge curve, which is now a function of both $\theta$ and $\tilde{\varepsilon}$, shifts downward as explained in section 6.2 above. This in turn contributes to dampening the effect of $i$ on unemployment as illustrated in Figure 10. The increase in $i$, and the resulting downward shift in the MD curve, lowers LM tightness from point $\theta'$ to $\theta''$. Unemployment level $u''$ is the level that would occur keeping the Beveridge curve fixed. As the latter shifts downward, the resulting equilibrium level of unemployment is lower, $u''' < u''$ in Figure 10, but still higher than the initial level $u'$.

As shown in the proof of Proposition 6 (cf. Appendix E.6), the transmission mechanism from monetary policy to firms' entry and formalization decisions through money demand works only when firms have some market power; that is when the bargaining power of buyers $\varphi < 1$. In the DM pricing equation (8), when $\varphi = 1$ we have $g(q) = c(q)$ as buyers extract all the surplus and firms make zero profits. Revenues from sales are reduced to

$$R_e(\varepsilon) = R_i(\varepsilon) = \varepsilon y$$

as firms are now indifferent between selling in the DM or the CM. This results in a vertical long run Phillips curve as changes in $i$ have no effect on the entry of firms and unemployment. This does not mean that money is superneutral since DM consumption $q$ and hence welfare are still affected by changes in the growth rate of money. It just means that there is no transmission from monetary policy through the money demand channel to the supply side of the economy.

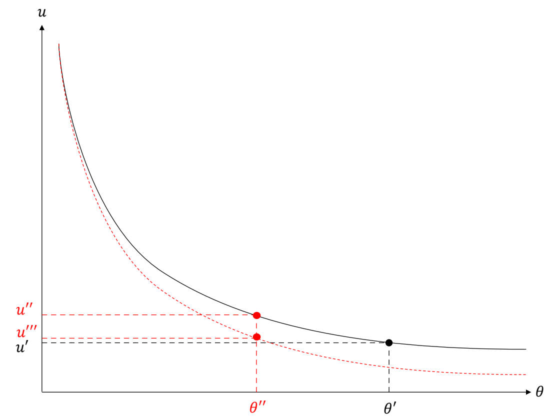



Figure 10: Effect of an increase in $i$ when $\delta_i > \delta_e$.

### 6.4 Incentive-feasible deflation

According to Proposition 5, the Friedman rule is necessary to achieve the socially optimal allocation given that the other efficiency conditions are satisfied. In most monetary models, the Friedman rule is implemented by extracting money from the economy in a lump-sum fashion in order to achieve deflation at the rate of time preferences. This results in a nominal interest rate $i = 0$. However, such a policy requires the capacity of the government to enforce potentially high levels of taxation. As shown by Andolfatto (2013) in the basic New-Monetarist model with limited commitment, deflation at the level required by the Friedman rule is not necessarily incentive-feasible. I revisit this issue in a context where taxation distorts the supply side by inducing firms and workers to operate informally. In particular, I assume here that the government can tax the economy only through $\tau_e$, a lump-sum tax on formal production. The money market clearing condition states that

$$\phi M = \phi m = z = g(q).$$

I use it to rewrite the government budget constraint (3) as

$$G + ub = T + n_e\tau_e + (\gamma - 1)g(q).$$

Assuming $G = T$ and $b = 0$ holds I get the following balanced budget rule:

$$0 = n_e\tau_e + (\gamma - 1)g(q).$$

which asserts that if the government is unable to levy additional lump-sum taxation, some form of distortionary taxes has to be used to implement the Friedman rule; $\tau_e$ in this case. The Friedman rule $i = 0$ requires $\gamma = \beta < 1$. This yields the following expression for $\tau_e^{FR}$:

$$\tau_e^{FR} = \frac{(1-\beta)g(q^*)[\alpha_h(\theta^{FR}) + \delta]}{\alpha_h(\theta^{FR})[1 - F(\tilde{\varepsilon}^{FR})]}.$$

Replacing this in the informality threshold equation and evaluating at $i = 0$ yields

$$\tilde{\varepsilon}^{FR} = \frac{(1-\beta)g(q^*)[\alpha_h(\theta^{FR}) + \delta]}{\chi\alpha_h(\theta^{FR})[1 - F(\tilde{\varepsilon}^{FR})]y} - \frac{\sigma_f\,[g(q^*) - c(q^*)]}{y}.$$

On the one hand, taxing formal firms and workers reduces the incentive to operate in the formal sector and hence increases informality. On the other hand, as we know from Proposition 6, by making money costless to carry it increases profits from DM monetary trades and increases the incentive to operate in the informal sector which again increases informality. The Friedman rule is incentive-feasible as long as $\tilde{\varepsilon}^{FR} < \bar{\varepsilon}$. Assuming the Hosios condition holds in both LM and DM, it is clear from Proposition (5) that implementing the Friedman rule through a tax on the formal sector will not produce the socially optimal allocation unless unemployment is taxed. This is because the Friedman rule achieves the allocation $(\theta^{FR}, q^*)$ where $\theta^{FR} < \theta^*$. Achieving $\theta^{FR} = \theta^*$ requires $b < 0$. A tax on unemployment encourages firms' entry by lowering the outside option of workers which lowers wages and hence increases firms profits.

# 7 Quantitative analysis

The model presented in the previous sections offers a micro-founded theoretical framework to study frictional goods and labor markets in developing economies with a large informal sector. It provides several insights into the interaction between monetary policy, unemployment and informality. In what follows, I assess the quantitative performance of the model by taking it to the data through calibration and then conducting some numerical exercises. For that, I use the stochastic version of the model presented in appendix B.

## 7.1 Calibration

The model parameters are calibrated using Brazilian data.[24] Brazil has a sizable informal sector, close to the average level in Latin American countries (Perry et al., 2007), and relatively

[24]See Ayres et al. (2019) for a review of the monetary history of Brazil.

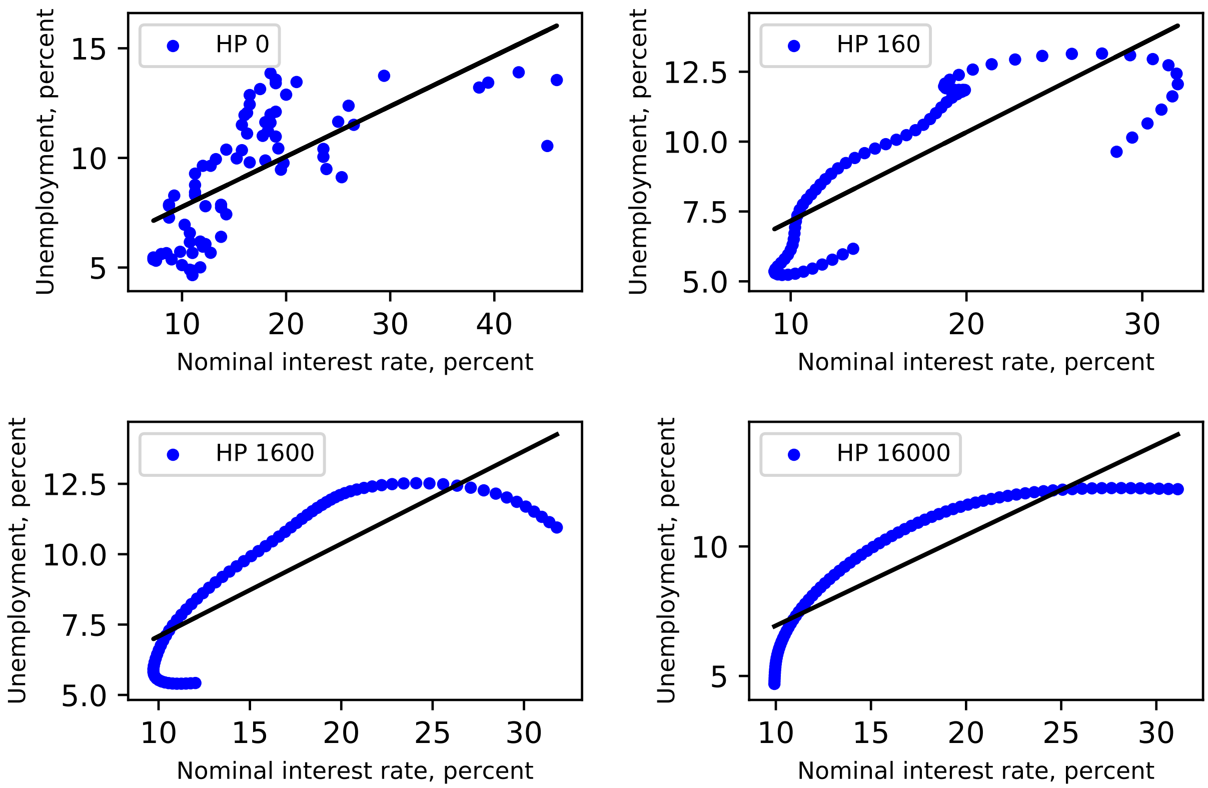



Figure 11: Nominal interest rates and unemployment in Brazil.

good statistics about unemployment that takes into account employment in the informal sector.[25] Each period in the model corresponds to a quarter. Limited data availability restricts the sample to the period from 1996Q3 to 2015Q3.[26]

Figure 11 depicts scatter plots of the nominal interest rate against unemployment. I gradually filter the data by removing high frequency fluctuations with increasingly higher values of the Hodrick–Prescott filter parameter. The data shows clearly a positive relationship between these variables at low frequency in line with the theory.

I choose the following functional forms: The utility function in the DM is $v(q) = Aq^{1-a}/(1-a)$. Utility in the CM is linear $U(x) = x$. Firms' cost function in the DM is linear $c(q) = q$ such that the marginal cost is the same as in the CM. In the absence of data on the productivity distribution of firms, I use a uniform distribution with support $[\underline{\varepsilon}, \bar{\varepsilon}]$ for the match productivity $\varepsilon$. The matching functions in the LM and DM are described

[25]The Brazilian labor force survey covers both formal and informal employment. See for example the discussion in Gerard and Gonzaga (2016).

[26]Unemployment data is limited by the change in the labor force survey methodology that occured in 2015 from the PME (Pesquisa Mensal de Emprego) to the new PNADC (Pesquisa Nacional de Amostras a Domicilio Contínua). I use PME data as it offers a longer although less recent data series.

Table 1: Independent parameters

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\beta$ | Discount factor | 0.98 | Data |
| $i$ | Nominal interest rate | 0.04 | Data |
| $y$ | General productivity level | 1.00 | Normalization |
| $C$ | DM cost function level | 1.00 | |
| $\zeta$ | DM cost function curvature | 1.00 | |
| $\eta$ | Credit availability in formal DM matches | 0.20 | % Credit card in retail transactions |
| $\delta$ | Job separation rate | 0.06 | Bosch and Esteban-Pretel (2012) |
| $\sigma$ | Elasticity of LM matching function | 0.34 | Meghir et al. (2015) |
| $\omega$ | LM workers bargaining power | 0.66 | Hosios condition |
| $F(\varepsilon)$ | Distribution of match productivities | Uniform [1,2] | - |
| $\rho_i$ | Autocorr. of interest rate shocks | 0.87 | Data |
| $\epsilon_i$ | SD of interest rate shocks | 0.04 | Data |
| $\rho_y$ | Autocorr. of labor productivity shocks | 0.95 | Data |
| $\epsilon_y$ | SD of labor productivity shocks | 0.01 | Data |

by $\mathcal{M}(u,v) = \xi u^{1-\sigma}v^{\sigma}$ and $\mathcal{N}(B,S) = BS/(B+S)$ respectively. As argued by Lazaryan and Lubik (2017) and others, in discrete time the LM matching probabilities $\alpha_h$ and $\alpha_f$ can take values above one. Intuitively, if the time period is long enough, everyone can exit unemployment at least once. This violates the idea that the matching in the labor market is probabilistic. Restricting the probabilities to lie on the unit interval requires

$$\left(\frac{1}{\xi}\right)^{1/(\sigma-1)} < \theta < \left(\frac{1}{\xi}\right)^{1/(1-\sigma)}$$

which is a non-empty interval for $\theta$ when $\xi \in (0,1)$. I impose this restriction in the calibration.

I separate the model's parameters into two groups: independent parameters and jointly calibrated parameters. The first group consists of parameters which I directly set to a specific value while the second group is jointly calibrated with the model's solution to match certain targets from the data as explained later.

The chosen values for the first group of parameters are listed in Table 1. $\beta$ is set such that the real interest in the model matches the average difference between the risk-free nominal interest rate and the rate of inflation in the data. The elasticity of the LM matching $\sigma$ is set to 0.34 based on the estimates of Meghir et al. (2015). The LM bargaining power of workers $\omega$ is set equal to $1-\sigma$ to satisfy the Hosios efficiency condition. I set $\delta$ to match the observed average quarterly job separation rate of formal and informal jobs in the data. Bosch and Esteban-Pretel (2012) reports rates of 3% and 10% respectively. Given the steady state composition of the labor force, I set $\delta$ at 5.72%. The share of credit formal matches $\eta$ is set at 20% which corresponds roughly to the average share of credit card transactions in total consumer spending transactions in Brazil.[27]

Once all the independent parameters are set I proceed to jointly calibrate the second group of parameters. This group comprises utility function parameters $A$ and $a$, DM bargaining power $\varphi$, the matching function efficiency $\xi$, the cost of posting vacancies $k$, the lump-sum tax on formal firms $\tau_e$, the probability of success of government monitoring $\chi$, the utility and pecuniary flow values of unemployment $b$ and $l$.

To specify the theoretical moments to match, I first define how the model maps into data. In terms of output I define

$$Y_{e_{DM}} = n_e \sigma_f \left[\eta(g(q_c) - c(q_c)) + (1-\eta)(g(q) - c(q))\right];$$

$$Y_{i_{DM}} = n_i(1-\chi)\sigma_f(g(q) - c(q));$$

$$Y_{e_{CM}} = n_e \frac{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon)}{F(\bar{\varepsilon}) - F(\tilde{\varepsilon})} + (1 - F(\tilde{\varepsilon}))u\theta k;$$

$$Y_{i_{CM}} = n_i(1-\chi) \frac{\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon y \, dF(\varepsilon)}{F(\tilde{\varepsilon}) - F(\underline{\varepsilon})} + F(\tilde{\varepsilon})u\theta k,$$

where $Y_{e_{DM}}$ and $Y_{i_{DM}}$ are the net aggregate output of formal and informal firms sold in the DM and $Y_{e_{CM}}$ and $Y_{i_{CM}}$ are the net aggregate output sold in the CM. Depending on the country, the real GDP in the data might take into account some informal activities (Andrews et al., 2011). In some countries, GDP accounts only for formal activities which is the case for example in the US. In some others, GDP includes formal and some of the informal activities. This is increasingly the case in EU countries. For the purposes of calibration I assume that all informal activities are included in Brazil's GDP data. To match that, I define GDP in the model as

$$Y = Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}} + Y_{i_{DM}}.$$

The size of the informal sector as a share of GDP is defined as

$$\frac{Y_i}{Y} = \frac{Y_{i_{DM}} + Y_{i_{CM}}}{Y}.$$

The nominal money supply $M$ corresponds in the model to the total amount of nominal money balances carried by households to be spent in both formal and informal monetary transactions in the DM. In the data, $M$ corresponds to either M1 or sweep-adjusted M1

[27] The figures are available on the website of the Associação Brasileira das Empresas de Cartões de Crédito e Serviços.

(Aruoba et al., 2011). I define the model-based money demand equation as

$$L(i) = \frac{M/p}{4 \times Y^{obs}} = \frac{g(q)}{4 \times Y}$$

where the demand for money $L(i)$ depends on $i$ directly through real balances $g(q)$ and indirectly through GDP.

Following Lucas (2000) and Lagos and Wright (2005), $A$ and $a$ are calibrated to fit $L(i)$ to the data. The idea is to match two moments: the average real money balances at the average nominal interest rate and the elasticity of money demand to the nominal interest rate.[28] To estimate the interest elasticity of money demand, I use a log-log specification

$$\log M_t/PY_t = \beta_1 + \beta_2 \log i_t + \nu_t.$$

The OLS estimate of $\beta_2$ is used as a point estimate for the interest elasticity of money demand $\epsilon$:

$$\epsilon = \frac{\partial L(i)}{\partial i} \frac{i}{L(i)}$$

Following Aruoba et al. (2011), I use $\varphi$, the bargaining power of buyers in the DM, to match the average markup in the economy. The markup in monetary transactions is defined as $\mu_m = \frac{g(q)/q}{c'(q)} - 1$ and in credit transactions as $\mu_c = \frac{g(q_c)/q_c}{c'(q_c)} - 1$. The average markup in DM trades is

$$\mu_{DM} = \frac{n_e(\eta\mu_c + (1-\eta)\mu_m) + n_i(1-\chi)\mu_m}{n_e + (1-\chi)n_i}.$$

Since the CM is a competitive market, the markup there is 0. Taking the average, I get $\mu = \mu_{DM}\frac{Y_{e_{DM}} + Y_{i_{DM}}}{Y}$. De Loecker and Eeckhout (2018) reports a net markup of around 61% for Brazil in 2016. According to their estimates, this value remained fairly stable since 1980. I match the above expression to target their number.

I match the steady state unemployment level to the average unemployment rate in Brazil over the period 1996-2015 which was around 9.35%. I match also the share of informal employment in total employment. The annual average reported by the International Labor Organization based on the Brazilian labor force survey (PNAD) over the period 2009-2013 stood at 38.9% of non-agricultural employment. In addition, I calibrate the model such that $\theta$ is normalized to 1 as there is no available data on vacancy posting or the tightness of the Brazilian labor market

The World Bank's Doing Business reports an average tax rate of 68% of before-tax

[28] An alternative is to solve for the parameter values which minimize the distance between the model-based money demand and the observed money demand for each observed interest rate. However, this procedure is computationally more consuming and doesn't affect much results.

Table 2: Calibration results

| | | Data | Model |
|---|---|---|---|
| **Jointly calibrated parameters** | | | |
| $k$ | Cost of vacancy posting | - | 0.02 |
| $\xi$ | LM matching efficiency | - | 0.55 |
| $\tau_e$ | Tax on formal production | - | 0.14 |
| $\chi$ | Probability of gov. monitoring | - | 0.08 |
| $\varphi$ | Bargaining power in DM | - | 0.46 |
| $A$ | DM utility level | - | 1.16 |
| $a$ | DM utility curvature | - | 0.43 |
| $b$ | Unemployment benefits | - | 0.89 |
| $l$ | Value of leisure | - | 0.71 |
| **Calibration targets** | | | |
| $u$ | Steady state unemployment rate | 9.35% | 9.35% |
| $n_i/(1-u)$ | Informal empl. in tot. empl. | 38.9% | 38.9% |
| $\theta$ | Normalization of LM tightness | - | 1.0 |
| $b/\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} w_e(\varepsilon)\frac{dF(\varepsilon)}{F(\bar{\varepsilon})-F(\tilde{\varepsilon})}$ | UI replacement rate | 50% | 50% |
| $\tau_e/\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon)\frac{dF(\varepsilon)}{F(\bar{\varepsilon})-F(\tilde{\varepsilon})}$ | % of tax burden in gross profits | 68% | 68% |
| $\mu_{DM}$ | Average DM markup | 61% | 61% |
| $L(i)$ | Average Money Demand | 22.18% | 22.30% |
| $\epsilon$ | Elasticity of MD to $i$ | -0.335 | -0.369 |
| $\sigma_u/\sigma_y$ | Rel. vol. of unemployment rate | 5.54 | 5.27 |

commercial profits in Brazil in 2016. This measure includes all taxes and mandatory contributions payable by the firm after accounting for allowable deductions and exemptions.[29] In the model, the average tax burden on formal corresponds to

$$\frac{\tau_e}{\frac{1}{F(\bar{\varepsilon})-F(\tilde{\varepsilon})}\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon)dF(\varepsilon)}.$$

I set $\tau_e$ to match a value of 68%.

As discussed by Ljungqvist and Sargent (2017), the calibration of the flow value of unemployment, $b+l$, which captures the value of leisure, unemployment benefits and home production, is very important for the quantitative performance of the model as it determines the elasticity of vacancy posting and hence unemployment to changes in productivity or other aspects that affect the return on job creation. This is because it determines, mostly, the fraction of resources that the market can allocate to vacancy creation, what the authors call the fundamental surplus fraction. The literature has followed different paths when it

[29]These taxes include the corporate income tax, all social contributions and labor taxes paid by the employer, property taxes, turnover taxes and other taxes. However, this measure excludes the value-added tax (VAT) which does not affect formal firms profits but is effectively a tax on formal goods consumption.

comes to the calibration of $b + l$. The standard one is to target a given percentage of the average productivity. Shimer (2005) sets the value to 40%, Hall and Milgrom (2008) choose 0.71 while Hagedorn and Manovskii (2008) argue for a value of around 96%. Instead, I separate $b$ and $l$ and proceed as follows: Unemployment insurance benefit $b$ is calibrated such that the average replacement rate in the model matches the empirical replacement rate. In Brazil, the benefit level ranges between 100% to 187% of the minimum wage which means that the replacement rate is very high for formal workers at the bottom of the wage distribution (Gerard and Gonzaga, 2016). These benefits are funded by a 0.65% sales tax on formal firms and only received by unemployed and informal workers who were formally employed. Gerard and Naritomi (2019) report an average replacement rate of around 70% over the period 2011-2013. Their sample however covers only urban workers. Hijzen (2011) reports a value of around 55% in 2009. I choose to match a gross replacement rate of 50% of the average wage in the formal sector such that

$$\frac{b}{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} w_e(\varepsilon) dF(\varepsilon)/(1 - F(\tilde{\varepsilon}))} = 50\%.$$

For the value of leisure $l$, I follow the procedure adopted by Berentsen et al. (2011) by calibrating $l$ to match the relative volatility of unemployment to output observed in the data as explained below.

The joint calibration procedure consists in two steps: First, given an initial value for $l$ I solve for the vector of calibrated parameters $\mathcal{P} = \{A, a, \varphi, k, \xi, \tau_e, \chi, b\}$ and the equilibrium solution vector $X$ which together reduce the distance (squared percentage difference) between the targeted moments $S_{data}$ and the corresponding theoretical moments $S_{model}$ subject to the system of steady state equilibrium equations as equality constraints:

$$\begin{aligned} \min_{\mathcal{P},X} \, & (\mathcal{S}_{model}(X; \mathcal{P}) - \mathcal{S}_{data})^2 \\ \text{s. t.} \quad & \mathbf{EC}(X; \mathcal{P}) = 0. \end{aligned}$$

Second, using the resulting parameter values from step 1 and the AR(1) processes for labor productivity and nominal interest rate shocks estimated from the data, I solve the stochastic version of the model using the algorithm described in Appendix C. I then run a series of simulations using 3'000 draws from the productivity shock process and calculate the resulting relative volatility of unemployment. If the simulated moment matches the empirical relative volatility of unemployment I stop. Otherwise I increase the value of $l$ and return to step 1.

Table 2 summarizes the results of the joint calibration procedure. The model matches well all of the targets including the interest elasticity of money demand and the ratio of the

Table 3: Model validation

| | | Data | Model |
|---|---|---|---|
| $Y_{DM}/Y$ | Share of DM in output | - | 16.0% |
| $\ell\%$ | Share of credit in DM sales (value) | 15.0% | 17.1% |
| $\mu$ | Economy average markup | 61% | 9.78% |
| $\bar{w}_e(\varepsilon)/\bar{w}_i(\varepsilon)$ | Ratio of average wages | - | 1.22 |
| $Y_i/Y_e$ | | - | 43.6% |
| $Y_i/(Y_e + Y_{iCM})$ | | - | 32.2% |
| $Y_i/Y$ | | 37.6% | 30.3% |

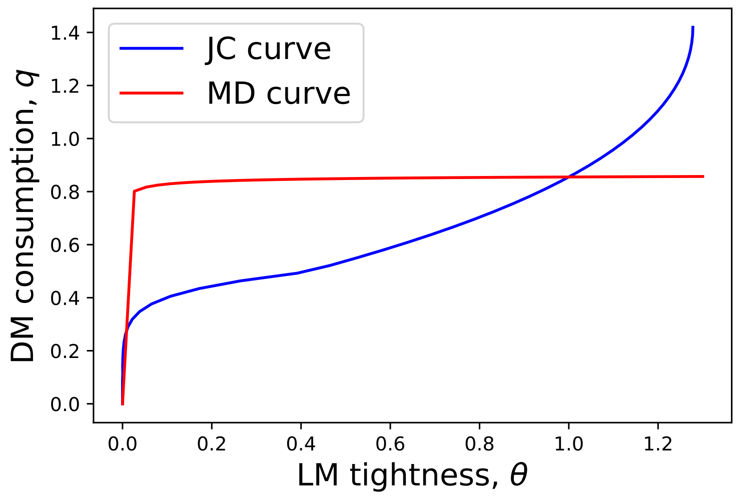



Figure 12: Steady state equilibria of the calibrated model

volatility of unemployment to output. In order to test the validity of the model I compare its calibrated equilibrium solution to empirical statistics that I have not included as targets. The size of the DM resulting from the calibration is 16.0% of total output. The ratio of DM to CM output in the informal sector is 0.23 compared to 0.17 in the formal sector. The average markup in the economy is around 10%. The share of credit transactions in the total value of DM transactions is 17.0%, very close to the one observed in the Brazilian retail sector.

In Table 3, I report as well the resulting values for the size of the informal sector. Depending on the measure used, the model generates values that are close to the reduced-form estimate of the size of the informal sector in Brazil. Notably, Medina and Schneider (2017) report an estimated average size of the informal sector in Brazil of 37.63% with a minimum of 32.56% and a maximum of 41.69% over the period 1991-2015 using the MIMIC (multiple indicators-multiple causes) model, a linear latent variables econometric approach.

Figure 12 plots the JC and MD curves resulting from the calibrated model. As predicted

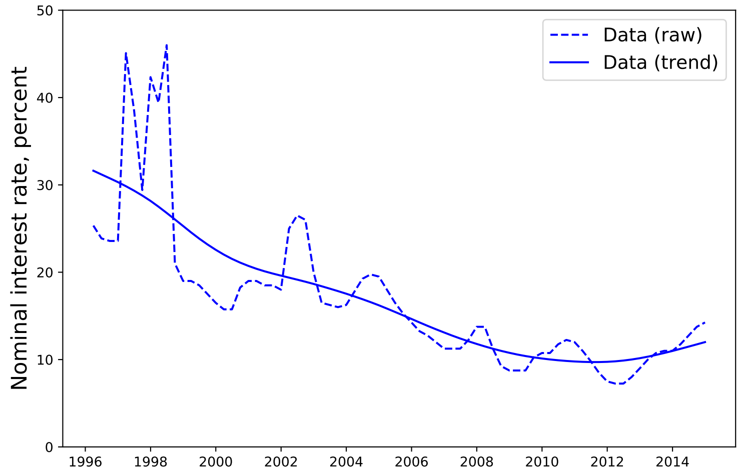



Figure 13: Nominal interest rates in Brazil, percent

by the theory, the model exhibits multiple stationary equilibria. In particular, two monetary equilibria one with high entry and real balances and the second with low entry and real balances. The calibrated equilibrium corresponds to the equilibrium.

## 7.2 Quantifying theory

Using the calibrated parameters, I input the actual time series of the nominal interest rate $i$ and compute the implied behavior of the model's variables holding productivity $y$ constant. By comparing the simulated series of money demand and unemployment with the actual time series I can assess how much of the long run movements can be attributed solely to changes in monetary policy. Figure 13 depicts the actual unfiltered and HP filtered time series of nominal interest rates in Brazil over the period 1996-2015 used to generate the counterfactual time series. The nominal interest rate spikes at the beginning of the period then trends downwards with some minor fluctuations.

Figure 14 plots both the actual and counterfactual time series for the ratio of M1 to nominal GDP, i.e. real money balances per unit of output. The model does a very good job in matching the observed increase in $M1/PY$ up until 2008 and the following decrease. In line with the theory, the model is better at matching the low frequency component as opposed to the raw data. This can also be seen from the ratio of standard deviations of real balances reported in Table 4. The model-implied data matches perfectly the volatility of real balances in the low frequency observed data and to a lesser extent the raw data. Notice that actual real balances react to both fluctuations in interest rates as well as aggregate productivity which is held constant in the counterfactual time series.

Next, I turn to the behavior of unemployment. Figure 16 depicts the actual and coun-

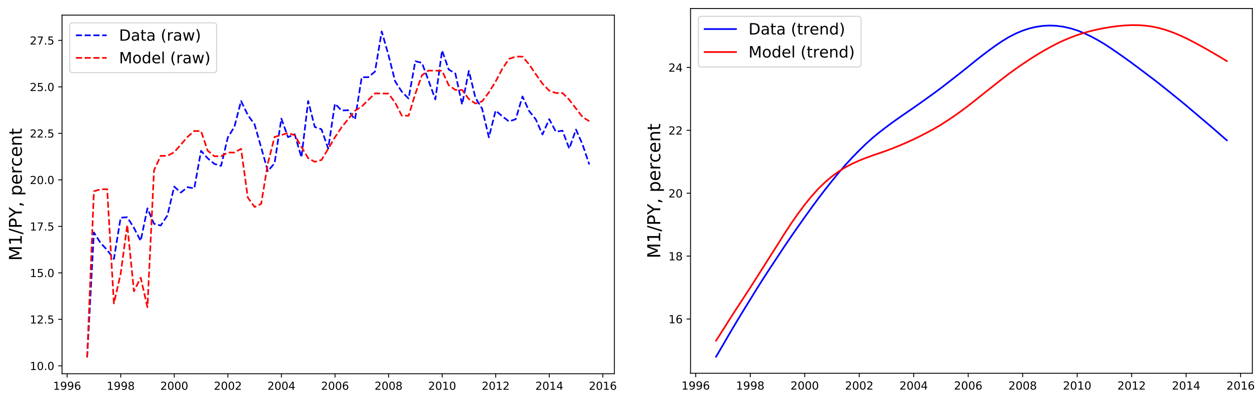



Figure 14: Real money balances: model vs. data.

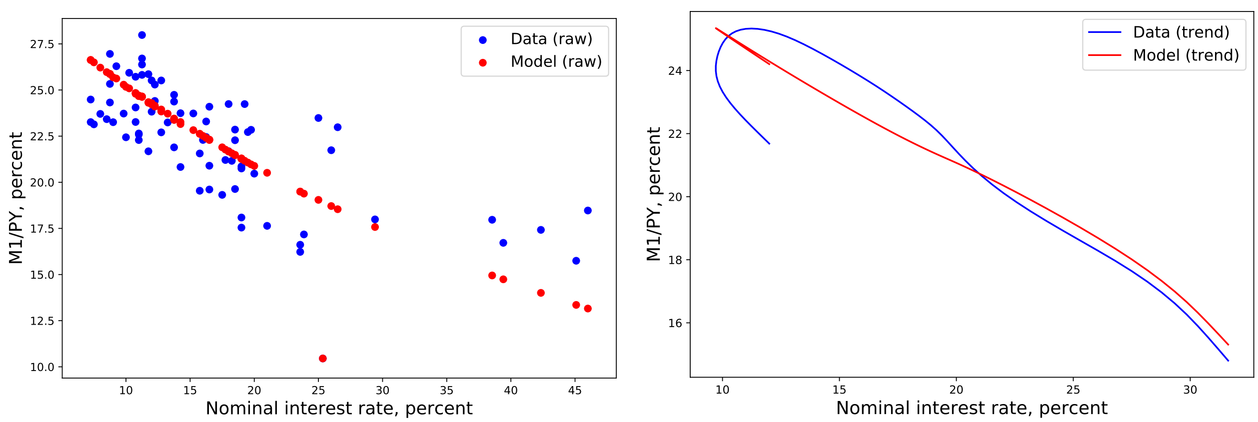



Figure 15: Money demand relationship: model vs. data.

terfactual time series of the unemployment rate $u$. The $u$ implied by the model is able to match around half of the spike observed between 1996 and 1999. Although to a lesser extent, the implied $u$ tracks also the increase observed between 2000 and 2003. However, the model fails to replicate the strong fall in unemployment that started in 2006.

Using a different perspective, figure 17 shows the long run Phillips curve, i.e. the scatterplot of $i$ against $u$, for both actual and simulated data. The model is able to match the upward sloping pattern of the actual Phillips curve. However the shape of the counterfactual curve is more dampened in comparison to the empirical curve. This is in part a result of the inability of the model to replicate the fall in unemployment observed in the last decade of the considered period.

Table 4 shows the ratios of standard deviations of simulated and actual data for unemployment and real money balances. The model is able to capture around a quarter of the volatility observed in trend unemployment while it capture all of the volatility in trend real balances.

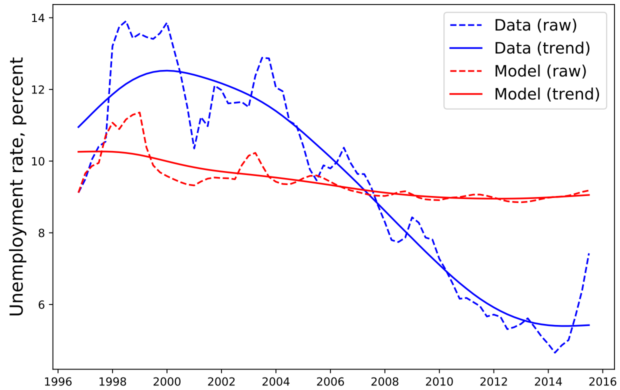



Figure 16: Unemployment rate: model vs data.

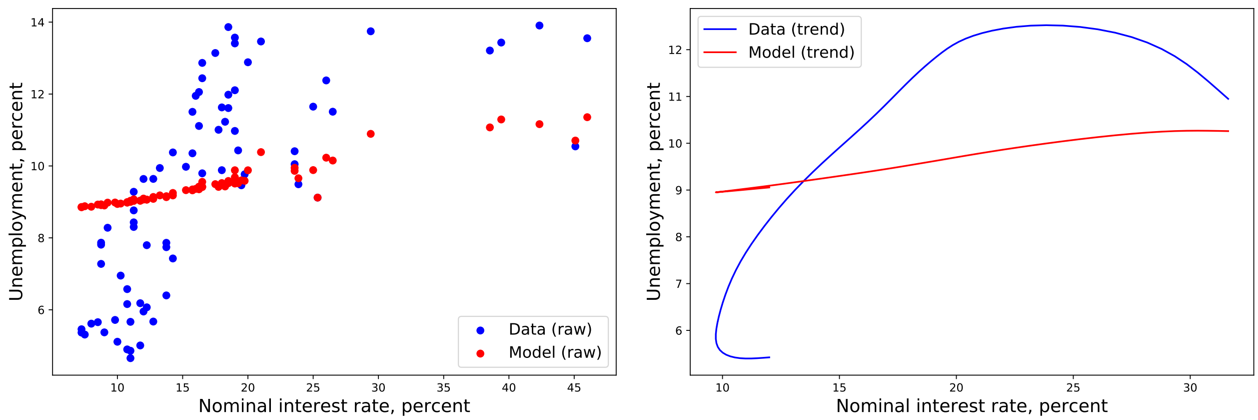



Figure 17: Long run Phillips curve: model vs data.

What are the implications for informal employment? The left panel of figure 18 depicts the model-implied time series.[30] One can clearly see that the downward trend in the nominal interest rate results in an increase in informal employment. In addition, the ratio of informal to formal employment follows the same pattern as changes in informal employment are more pronounced compared to formal employment. This is a result of the informal firms being more affected by changes in the opportunity cost of money.

## 7.3 Cost of inflation

Table 5 reports changes in the steady state equilibrium allocations resulting from increasing the inflation rate from the Friedman rule (i.e. $\gamma = \beta$) to a 10% annual rate. This corresponds to an increase in the annual nominal interest rate from $i = 0$ to around 20%. As expected,

[30] Since there is no available statistics on informal employment in 1996Q3, I initialize the time series by setting informal employment at its steady state level. Unemployment is initialized using its actual value in 1996Q3.

Table 4: Ratios of standard deviations of model simulations to data

| | Unfiltered | Trend |
|---|---|---|
| Unemployment | 0.21 | 0.17 |
| $M/PY$ | 1.06 | 0.99 |

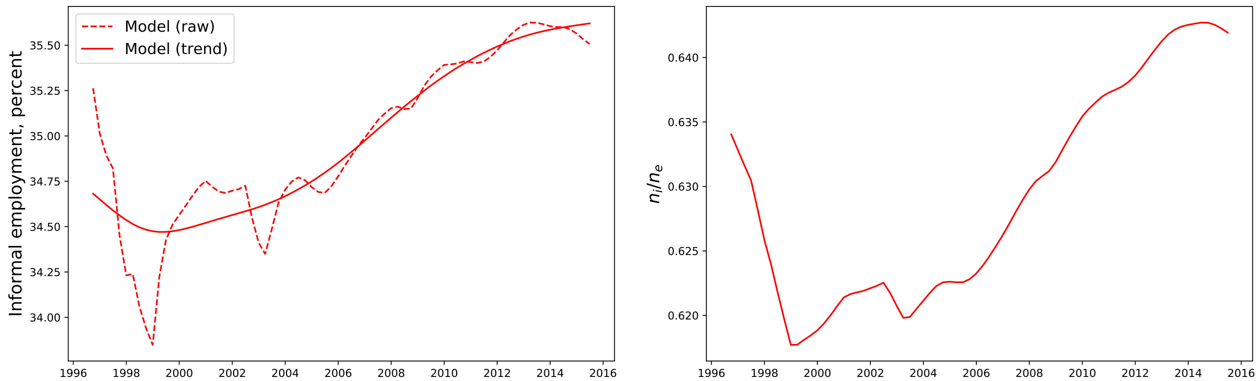



Figure 18: Model-implied informal employment.

DM consumption in monetary matches decreases drastically, unemployment increases by 1.2pp and informal employment decreases by 1.7pp. Formal employment increases only by 0.5pp as some of the job creation shifts away from the informal sector to the formal sector. Aggregate output decreases by almost 3% as informal output falls by around 6%.

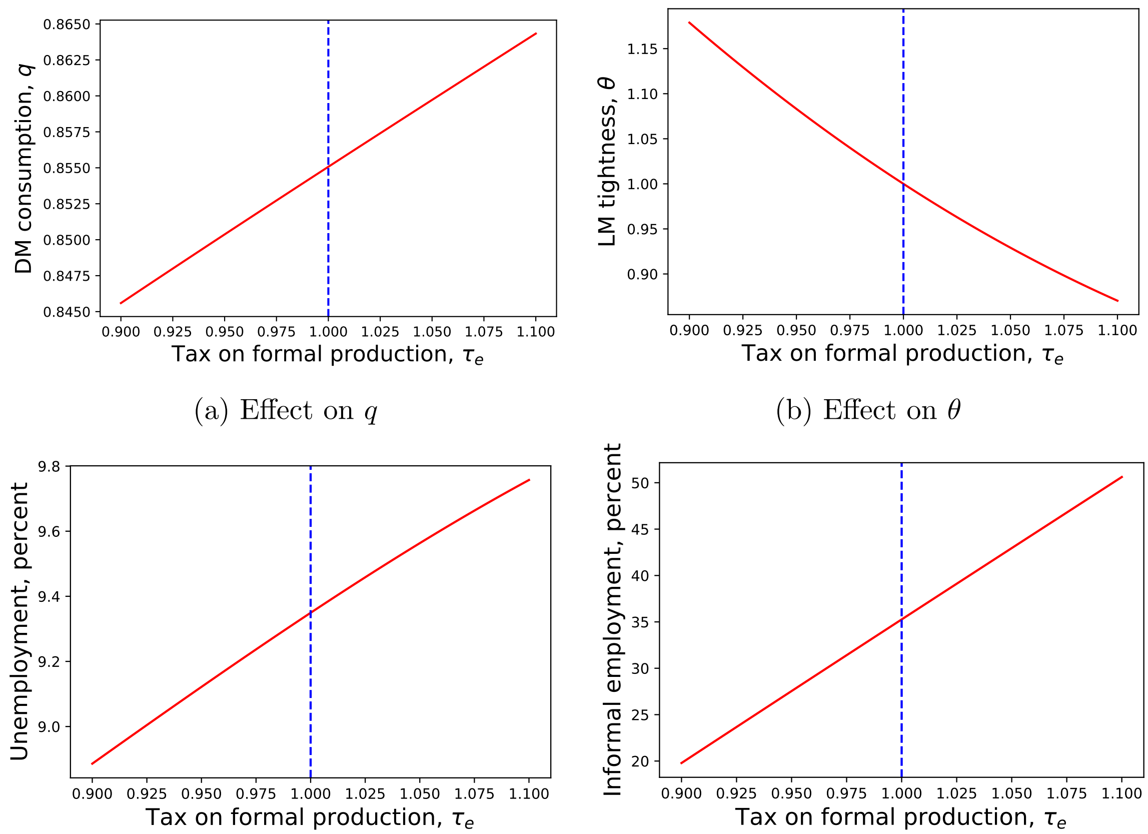
This effect of inflation is non-linear. Increasing annual inflation from 10% to 20% reduces $q$ by roughly the same percentage but the effect on unemployment is 30% stronger as seen in the second line of Table 5. This differential effect on $q$ and $u$ hints that the non-linearity comes from frictions in the LM. A similar result has been highlighted by Petrosky-Nadeau and Zhang (2016) who argue that the concavity of the LM matching function results in a significant non-linearity in the sense that unemployment reacts differently to negative and positive shocks of the same magnitude. This can make recessions fast and severe while spreading recoveries over time.

## 7.4 Fiscal implications

To better understand the fiscal consequences of monetary policy, I first look at the effect of changes in the tax burden on the formal sector. The experiment consists in varying the calibrated value of $\tau_e$ (normalized to 1). As seen in Figure 19, increasing the tax burden has two effects: First, it lowers profits for firms and reduces entry and job creation which increases unemployment. Second, it drives firms to the informal economy where cash is the only means of payments. This results in an increase in money demand and the quantity

Table 5: Effect of increasing annual inflation rate from FR to 0%, 10%

| $\Delta i$ | $\Delta q$ | $\Delta u$ | $\Delta n_i$ | $\Delta n_e$ | $\Delta Y$ | $\Delta Y_e$ | $\Delta Y_i$ |
|---|---|---|---|---|---|---|---|
| $i = 0\%$ to $10\%$ | -30.32% | 0.30 pp | -0.47 pp | 0.17 pp | -0.80% | 0.03% | -1.84% |
| $i = 10\%$ to $20\%$ | -31.36% | 0.92 pp | -1.22 pp | 0.31 pp | -2.17% | -0.15% | -4.72% |




(a) Effect on $q$ (b) Effect on $\theta$

(c) Effect on $u$ (d) Effect on $n_i$

Figure 19: Steady state effects of changing $\tau_e$.

traded in DM monetary matches. The combined effect of lower entry and higher informality leads to a decrease in the size of the formal sector.

Next, I revisit the effects of changing the long run inflation rate from the previous section by taking into account the fiscal consequences through the government budget constraint. The money market clearing condition states that

$$\phi M = \phi m = z = g(q)$$

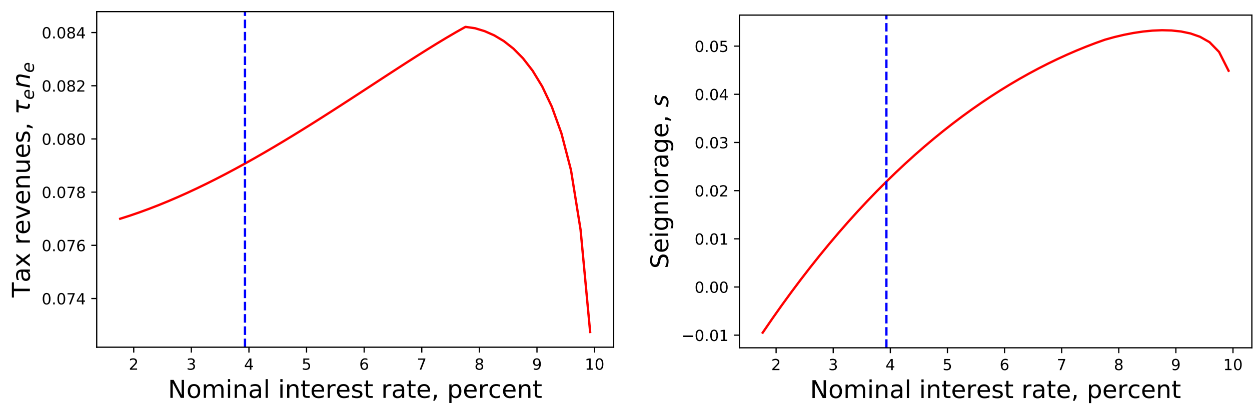



Figure 20: Steady state effects of $\gamma$ on government revenues.

Table 6: Steady state effect of increasing $\gamma$ on government revenues

| $\Delta i$ | $\Delta n_e \tau_e$ | $\Delta s$ | $\Delta bu$ |
|---|---|---|---|
| $i = 0\%$ to $20\%$ | 4.98% | -168.39% | 12.07% |
| $i = 20\%$ to $30\%$ | 3.99% | 54.87% | 17.37% |

which I use to rewrite the government budget constraint (3) as

$$G + ub = T + n_e \tau_e + s$$

where

$$s \equiv (\gamma - 1)\phi M = (\gamma - 1)g(q)$$

is the seigniorage income the government makes by increasing the supply of money at rate $\gamma$.[31] Table 6 depicts changes in government revenues and expenditures as a consequence of increasing $\gamma$ under the maintained assumption of adjustment through lump-sum transfers $T$. In particular, increasing $\gamma$ affects the government budget in three ways: First, it affects the size of the formal sector $n_e$ which in turn affects the revenues from the tax on formal production $\tau_e n_e$. Second, it generates additional seigniorage income the government can use to purchase goods and services or transfer to households. Third, it affects job creation and unemployment which determines the total amount of unemployment insurance (UI) benefits the government is redistributing to the unemployed.

To better understand these mechanisms, I gradually drop the assumption of lump sum taxes by imposing different balanced-budget rules and then study the implications of each

[31] I assume here that the government is able to collect seigniorage revenues on all the money supply. Since I calibrated the model to match M1, this assumption exagerates seigniorage revenues as in practice governments earn seigniorage only on M0 or the monetary base (M0 plus reserves).

on the equilibrium allocation.

**Rule 1:** The first rule assumes that the tax burden on formal firms finances a fixed amount $G$ such that

$$G = \tau_e n_e$$

which means that $\tau_e$ adjusts in response to changes in the size of the informal sector

$$\tau_e = \frac{G}{n_e}.$$

The first two lines of Table 7 show how the equilibrium allocation changes following an increase in $\gamma$ under Rule 1. We know that $n_e$ is increasing in $\gamma$ under the baseline calibration as seen in Table 5. Under Rule 1, as $\gamma$ increases, the size of the tax base increases and the amount of taxes per formal match $\tau_e$ decreases. This in turn amplifies the effect of inflation on informality compared to the case without a balanced budget rule. In particular, going from the Friedman rule to a 10% nominal interest rate decreases informal employment by 0.51pp under Rule 1 compared to the baseline experiment of 0.47pp. Formal employment increases by 0.22pp compared to 0.17pp. The additional effect of the reduced tax burden on the formalization margin actually doubles when going from a 10% to a 20% nominal interest rate. However, the decrease in the tax burden is not enough to compensate for the negative effect of inflation on entry and as such the effect on $u$ is almost the same as in the baseline experiment. This is also clear from looking at output as aggregate output stays almost the same while the reallocation from informal to formal output is larger compared to the baseline case without a balanced budget rule.

**Rule 2:** The second balanced budget rule dictates that both formal sector taxes and seigniorage income finance a fixed amount $G$ i.e.

$$G = \tau_e n_e + s$$

This implies that $\tau_e$ is a function of both seigniorage income and the size of the formal sector

$$\tau_e = \frac{G - s}{n_e}.$$

The allocations resulting from a higher inflation rate under Rule 2 are depicted in the third and fourth lines of Table 7. Once seigniorage income is taken into account, the previous results are overturned. Higher inflation now leads to a decrease in unemployment, an increase in output and a much larger reallocation of firms and jobs from the informal to the formal sector. This is the case as higher inflation simultaneously taxes the informal sector and uses the resulting seigniorage income to reduce the tax burden on the formal sector.

**Rule 3:** The third balanced-budget rule is the most general and dictates that formal sector taxes and seigniorage income finance UI benefits in addition to a fixed amount $G$ such that

$$G + bu = \tau_e n_e + s$$

which implies that either $b$ or $\tau_e$ adjust to balance the budget. I assume that $\tau_e$ adjust such that

$$\tau_e = \frac{G + bu - s}{n_e}.$$

The last two rows of Table 7 present the results of increasing the inflation rate under Rule 3. The main message here is that additional seigniorage income is not enough to compensate for negative effects of inflation on the extensive margin. This is because the resulting increase in unemployment leads to higher spending on unemployment insurance benefits. The additional seigniorage income is not enough to pay for that and hence $\tau_e$ has to increase. This leads to further increase in unemployment and a shift from the formal to the informal sector which exacerbates the shrinkage of the tax base and pushes $\tau_e$ higher.

This result is related to Rocheteau (1999) who shows that introducing a balanced-budget rule in the form of Rule 3 in the standard labor search model can result in multiple equilibria as different levels of unemployment are compatible with different levels of taxation when unemployment benefits are constant. To get rid of this multiplicity, the government should fix the overall spending on unemployment benefits $bu$ and allow $b$ to adjust as $u$ changes. This is similar to Rule 2 where the tax rate is independent of unemployment insurance benefits.

## 7.5 Optimal inflation rate

The government problem is to choose a gross inflation rate $\gamma$ in order to maximize social welfare subject to the monetary equilibrium conditions as well as its own balanced-budget constraint. I focus here on the steady state equilibrium with full commitment. The govern-

Table 7: Steady state effect of increasing $\gamma$ under a balanced budget rule

| $\Delta i$ | $\Delta q$ | $\Delta u$ | $\Delta n_i$ | $\Delta n_e$ | $\Delta Y$ | $\Delta Y_e$ | $\Delta Y_i$ |
|---|---|---|---|---|---|---|---|
| **Rule 1** | | | | | | | |
| $i = 0\%$ to $10\%$ | -30.31% | 0.29 pp | -0.51 pp | 0.22 pp | -0.79% | 0.15% | -1.96% |
| $i = 10\%$ to $20\%$ | -31.36% | 0.90 pp | -1.30 pp | 0.40 pp | -2.15% | 0.05% | -4.93% |
| **Rule 2** | | | | | | | |
| $i = 0\%$ to $10\%$ | | | | | | | |
| $i = 10\%$ to $20\%$ | -32.20% | -0.71 pp | -10.23 pp | 10.94 pp | 1.42% | 25.26% | -29.12% |
| **Rule 3** | | | | | | | |
| $i = 0\%$ to $10\%$ | -31.10% | 0.04 pp | 0.01 pp | -0.05 pp | -0.57% | -0.55% | -0.63% |
| $i = 10\%$ to $20\%$ | -31.68% | 0.10 pp | 1.27 pp | -1.37 pp | -1.76% | -3.71% | 3.25% |

ment solves:

$$
\begin{aligned}
(1-\beta)\mathcal{W} = \max_{\gamma \geq \beta} \Bigg\{ & -\frac{\theta u k}{\beta} + u(b+l) + n_e\Big[\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \varepsilon y \, dF(\varepsilon) - \tau_e\Big] \\
& + \sigma_h \frac{n_e}{1-u}\{\eta[v(q_c) - c(q_c)] + (1-\eta)[v(q) - c(q)]\} \\
& + n_i(1-\chi)\Big[\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon y \, dF(\varepsilon)\Big] + \sigma_h \frac{n_i}{1-u}(1-\chi)[v(q) - c(q)] \Bigg\}
\end{aligned}
$$

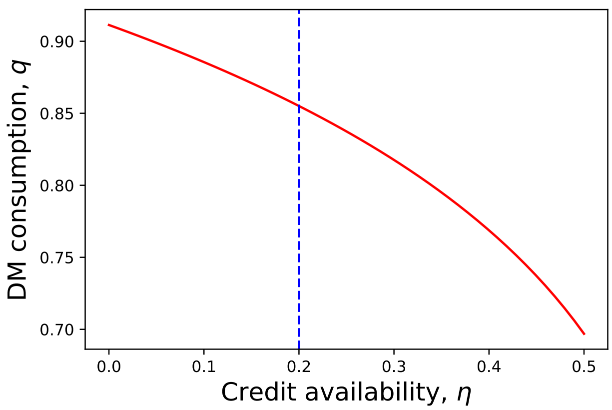
subject to the model's equilibrium conditions defined in Proposition 1 and balanced-budget Rule 1, 2 or 3 discussed in the previous section.

TBC!

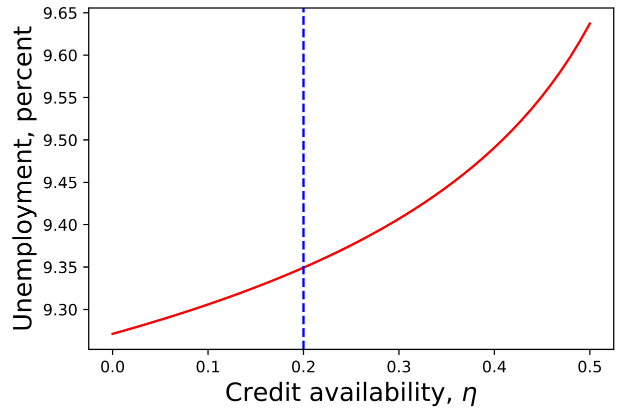
## 7.6 Financial development

How does financial development in the form of a wider availability of credit in the formal sector affect the economy? An increase in $\eta$ reduces $q$ for $i > 0$. This is because as credit becomes more available in the formal sector the share of transactions where money is needed for settlement will decrease and hence the demand for it will fall. This has two opposite effects on formal firms: First, it increases the frequency of transactions where credit is available which rises expected revenues. Second, it reduces the quantity traded in the remaining monetary matches which lowers expected revenues. The net outcome depends on which effect dominates. In contrast, the effect on informal firms goes only in one direction: higher $\eta$ lowers $z$ and $q$ and hence reduces the profits of informal firms. As such, credit availability and informality go in opposite directions. However, the net effect on firms' entry and unemployment is ambiguous.

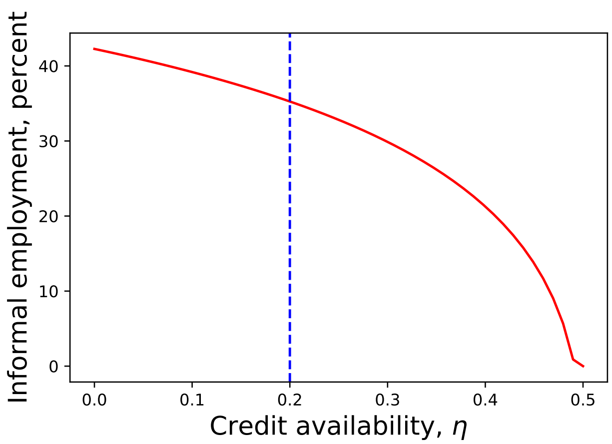
Figure 21 depicts the general equilibrium effects of higher $\eta$ under the baseline calibration.



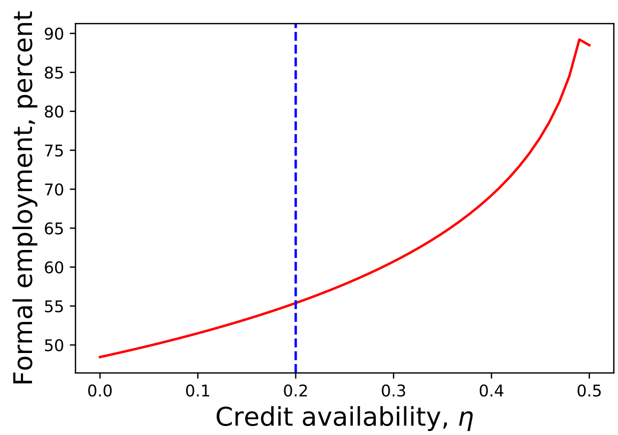

(a) Effect on $q$




(b) Effect on $u$




(c) Effect on $n_i$




(d) Effect on $n_e$

Figure 21: Steady state effects of changing $\eta$.

One can see that the negative effect on unemployment dominates but is negligible. In particular, increasing $\eta$ from 20% to 80% drives unemployment from 9.35% to 9.9%. However, the effect on informal employment is very strong as it falls from 45.1% to less than 30% of the labor force. Interestingly enough, formal employment increases from around 50% to more than 60% as the number of firms that switch from the informal to the formal sector more than compensates for the fall in firms' entry.

# 8 Extensions

In this section I consider various extensions to the baseline model and discuss their implications on the main results of the paper.

## 8.1 Payroll tax

In the presence of a proportional wage tax $\tau_w$, the sharing rule in formal jobs changes to

$$\omega V_e^f(\varepsilon) = (1+\tau_w)(1-\omega)\left(W_e^h(\varepsilon) - W_u^h\right)$$

In this case identical Nash bargaining power for workers in both types of jobs is not enough to guarantee identical surplus sharing rules. The marginal payroll tax reduces the match surplus and creates a wedge which distorts the way the surplus is split between the worker and the firm. Furthermore, since $\tau_w$ is proportional to the wage, the worker and the firm might find it optimal to reduce the wage in order to increase the total surplus of the match. The presence of a wage tax results also in a difference in sharing rules under formal and informal jobs which creates a discontinuity in the way the surplus is shared. This rises the possibility that for the same productivity level the firm and worker disagree on what is the optimal choice of contract. Intuitively, going from an informal to a formal contract for the same productivity level might simultaneously change the total surplus and decrease the share of one of the parties.
TBC!

## 8.2 Costly credit

In the baseline model, I assumed that credit in formal matches is costless. The coexistence of costless credit and money is a longstanding challenge for monetary economics. As discussed in Gu et al. (2016), if credit is easy money is not essential. If credit is tight money becomes essential but credit is irrelevant. The coexistence of credit and money in my model relies on the existence of informality. Some agents relinquish the use of credit because they want to avoid taxes and other costly government regulation. In this section, I discuss some of the implications of making credit costly. Following Bethune et al. (2019), I add a transaction cost for the use of credit. In particular, I assume a cost function $\zeta$ satisfying $\zeta(0) = \zeta'(0) = 0$ and $\zeta'(q), \zeta''(q) > 0 \; \forall q > 0$. To simplify I assume $\eta = 1$ and that the transaction cost is born by buyers.

As before, $q_c$ is the quantity consumed in formal credit matches and $q$ is the quantity consumed in informal monetary matches. The surplus of buyer in a credit match is now given by $v(q_c) - \phi p_c - \zeta(\phi p_c - d_c)$ where $p_c$ is the sum of payments in both credit and real balances. The surplus of the seller is given by $p_c - c(q_c)$.

The proportional bargaining solution is given by maximizing the buyer's surplus subject to the seller obtaining a proportion $\varphi$ of the total surplus:

$$\max_{q_c,p_c,d} v(q_c) - \phi p_c - \eta(\phi p_c - d_c)$$

subject to the liquidity constraint

$$d_c \leq z$$

and the seller's participation constraint

$$\phi p_c - c(q_c) = (1-\varphi)(v(q_c) - c(q_c) - \eta(\phi p_c - d_c)).$$

Given the real balances carried by the buyer $z$, the solution is a pair $(q_c, p_c)$ that satisfies the first order conditions

$$\frac{v'(q_c)}{c'(q_c)} = 1 + \eta(\phi p_c - z)$$

and

$$\phi p_c = (1-\varphi)(v(q_c) - \eta(\phi p_c - z)) + \varphi c(q_c),$$

where the amount borrowed by the buyer is $\phi\ell = \phi p_c - z$ at cost $\eta(\phi\ell)$.

Given the terms of trade in monetary and credit matches, the buyer decides on how much real balances to carry out of the CM by solving

$$\max_{z} (\beta - \gamma)z + \beta(\sigma_e(v(q_c) - \phi p_c - \eta(\phi p_c - z)) + \sigma_i(v(g^{-1}(z) - z)))$$

which yields the first order condition

$$i = \sigma_e \left( v'(q_c)\frac{dq_c}{dz} - \phi\frac{dp_c}{dz} - (\phi\frac{dp_c}{dz} - 1)\eta'(\phi p_c - z) \right) + \sigma_i \left( \frac{v'(q)}{g'(q)} - 1 \right).$$

Using the implicit function theorem, $\frac{dq_c}{dz}$ and $\frac{dp_c}{dz}$ can be derived from the bargaining solution to get

$$\frac{dq_c}{dz} = \frac{(1 + (\phi\frac{dp_c}{dz} - 1)\eta''(\phi p_c - z))(c'(q_c))^2}{v''(q_c)c'(q_c) - v'(q_c)c''(q_c)}$$

and

$$\frac{dp_c}{dz} = \frac{((1-\varphi)v'(q_c) + \varphi c'(q_c))\frac{dq_c}{dz} + (1-\varphi)\eta'(\phi p_c - z)}{\phi + \phi(1-\varphi)\eta'(\phi p_c - z)}.$$

Assuming $c(q) = q$ and a take it or leave it offer by the buyer (i.e. $\varphi = 1$), one gets:

$$\phi \frac{dp_c}{dz} = \frac{dq_c}{dz} = \frac{1 - \eta''(\phi p_c - z)}{v''(q_c) - \eta''(\phi p_c - z)} \geq 0$$

when $\eta''(\phi p_c - z) \leq 1$.

Assuming $c(q) = q$ simplifies the above to

$$\frac{dq_c}{dz} = \frac{1 + (\phi \frac{dp_c}{dz} - 1)\eta''(\phi p_c - z)}{v''(q_c)}$$

and

$$\frac{dp_c}{dz} = \frac{((1-\varphi)v'(q_c) + \varphi)\frac{dq_c}{dz} + (1-\varphi)\eta'(\phi p_c - z)}{\phi + \phi(1-\varphi)\eta'(\phi p_c - z)}.$$

TBC!

# 9 Conclusion

In this paper, I examine the macroeconomics of informality through the lens of a monetary dynamic general equilibrium model with flexible prices and frictional labor and goods markets. Informality in this model results from the optimal choices of firms and households given the frictions they face. A main innovation of this model is to connect informality in both labor and goods markets by limiting buyers to the use of money as a means of payment when trading with firms employing informal workers. The model can exhibit multiple stationary equilibria due to the strategic complementarity between households' money demand and firms' entry. I show that unemployment and informality can be negatively correlated across these equilibria.

Inflation affects real variables in the long run through a money demand channel as it taxes monetary balances carried by households for transaction purposes. The informal sector is more vulnerable to the inflation tax compared to the formal sector where credit is feasible as a means of payment. I show that an increase in the long run inflation rate reduces informality at the cost of higher unemployment. However, the net effect on the formal sector and tax revenues is ambiguous.

I calibrate a stochastic version of the model to the Brazilian economy and conduct a counterfactual analysis. In particular, I input the time series for the nominal interest rates and compare the implications for money demand and unemployment to the data. I also report the implied time series for informal employment. The model does a very good job in matching the low frequency movements in money demand observed in the data. The model is also able to match the low frequency behavior of unemployment in the first half of the sample period.

The model-implied time series for informal employment exhibits an upward trend as a consequence of the decrease in the opportunity cost of real balances observed in Brazil. One important policy implication of this is that reducing inflation might have the unintended consequence of increasing the size of the informal sector thus leading to higher unemployment and tax evasion. As a policy recommendation, reducing inflation should be accompanied by measures encouraging financial development in the formal sector to make it more attractive and hence counteract the resurgence of informality. This points to the importance of understanding and measuring informality for the implementation of monetary policy in developing economies in particular when choosing the long run inflation targets.

# Appendix A Model with different separation rates

In what follows I assume that formal and informal jobs are destroyed with exogenous probabilities $\delta_e$ and $\delta_i$ respectively with $\delta_i \geq \delta_e$ in line with the empirical evidence (cf. section 3). In this case, the measures of formal employment $n_e$, informal employment $n_i$ and unemployment $u$ evolve according to

$$\begin{aligned} u_{+1} &= (1-\alpha_h(\theta))u + \delta_e n_e + \delta_i n_i; \\ n_{e,+1} &= (1-\delta_e)n_e + \alpha_h(\theta)(1-F(\tilde{\varepsilon}))u; \\ n_{i,+1} &= (1-\delta_i)n_i + \alpha_h(\theta)F(\tilde{\varepsilon})u, \end{aligned}$$

subject to the condition $u + n_e + n_i = 1$. It follows that the steady state equilibrium in the labor market is given by the measures

$$u = \frac{\delta_e n_e + \delta_i n_i}{\alpha_h}; \quad n_e = \frac{\alpha_h(\theta)(1-F(\tilde{\varepsilon}))u}{\delta_e}; \quad n_i = \frac{\alpha_h(\theta)F(\tilde{\varepsilon})u}{\delta_i}.$$

From the system of equations above, I solve for $u$ as a function of $\theta$ and $\tilde{\varepsilon}$:

$$u = \frac{\delta_e + \delta_i \rho(\tilde{\varepsilon})}{(1+\rho(\tilde{\varepsilon}))\alpha_h(\theta) + \delta_e + \delta_i \rho(\tilde{\varepsilon})} \tag{A.1}$$

where

$$\rho(\tilde{\varepsilon}) \equiv \frac{n_i}{n_e} = \frac{\delta_e}{\delta_i}\frac{F(\tilde{\varepsilon})}{1-F(\tilde{\varepsilon})} \tag{A.2}$$

is the steady state ratio of informal to formal employment. $\rho$ is a function of the job separation rates, the match productivity distribution $F$ and the informality threshold $\tilde{\varepsilon}$.

Using $\tilde{\varepsilon}$ I rewrite equation (2) as the job creation (JC) condition

$$\begin{aligned} \frac{k}{\beta\alpha_f(\theta)} = &\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \frac{(1-\omega)\left(R_e(\varepsilon)-\tau_e-b\right)-\omega\theta k}{1-\beta(1-\delta_e)}\mathrm{d}F(\varepsilon) \\ &+ \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \frac{(1-\omega)\left((1-\chi)R_i(\varepsilon)-b\right)-\omega\theta k}{1-\beta(1-\delta_i)}\mathrm{d}F(\varepsilon). \end{aligned} \tag{A.3}$$

which determines firms' entry and the LM tightness $\theta$ and where $\tilde{\varepsilon}$ satisfies the productivity threshold condition (17). The MD equation is given by equation (23) but now $\rho(\tilde{\varepsilon})$ is given by (A.2) instead. The steady state equilibrium of this economy can be defined as follows:

**Definition 2** *A stationary monetary equilibrium in this economy is defined as: (i) a productivity threshold $\tilde{\varepsilon}$, (i) a level of LM tightness $\theta$, (iii) a level of unemployment $u$ , (iv) and quantities $\{q, q_c\}$ traded in the DM, which together satisfy*

- *Optimal consumption in credit and monetary DM matches* (**??**) *and* (23)*;*
- *The Job Creation equation* (A.3)*;*
- *The informality threshold equation* (D.2)*;*
- *The Beveridge curve* (A.1)*.*

# Appendix B  Stochastic Model

The state of the economy at the beginning of each period is $s = \{y, i, u, n_e, n_i\}$ where $u$ is unemployment, $n_e$ and $n_i$ measure formal and informal employment, $i$ the nominal interest rate and $y$ is the aggregate productivity level. The exogenous stochastic processes for the nominal interest rate and productivity are given by

$$i_{+1} = \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma_i)$$

$$y_{+1} = \bar{y} + \rho_y(y - \bar{y}) + \epsilon_y, \quad \epsilon_y \sim N(0, \sigma_y).$$

while labor market measures evolve according to the laws of motion

$$\begin{aligned} u_{+1} &= (1 - \alpha_h(\theta))u + \delta(1 - u); \\ n_{e,+1} &= (1 - \delta)n_e + \alpha_h(\theta)(1 - F(\tilde{\varepsilon}))u; \\ n_{i,+1} &= (1 - \delta)n_i + \alpha_h(\theta)F(\tilde{\varepsilon})u. \end{aligned} \tag{B.1}$$

The state of the economy next period $s'$ is known at the end of the CM of the current period. Given the realization of $y'$ and $i'$ and the current period labor force composition $\{u, n_e, n_i\}$, firms post vacancies for next period's LM. The resulting $\theta'$ and $\tilde{\varepsilon}'$ determine next period's labor force composition which in turn determines DM matching probabilities. There is no aggregate uncertainty for households in the sense that they decide on their real balances in the current CM after $y'$ and $i'$ are realized. This is not the case for firms as they make their entry and hiring decisions based on their expectations of future economic conditions. This is because firms' jobs might last several periods as opposed to households who get to rebalance their portfolio of money holdings every period.

In the LM, households' value functions are

$$U_u^h(0, z; s) = \alpha_h(s) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\left\{V_e^h(\varepsilon, z; s), V_i^h(\varepsilon, z; s)\right\} \, \mathrm{d}F(\varepsilon) + (1 - \alpha_h(s))V_u^h(0, z; s)$$

and

$$U_j^h(\varepsilon, z; s) = (1-\delta)V_j^h(\varepsilon, z; s) + \delta V_u^h(0, z; s), \quad j \in \{e, i\}.$$

In the DM, we have

$$\begin{aligned} V_j^h(\varepsilon, z; s) = {} & \sigma_e(s)\eta\left(v(q_c) - d_c\right) + \sigma_e(s)(1-\eta)\left(v(q(z;s)) - d(z;s)\right) \\ & + \sigma_i(s)\left(v(q(z;s)) - d(z;s)\right) + z + \mathbb{E}[W_j^h(\varepsilon, 0; s_{+1})],\ j \in \{u, e, i\}, \end{aligned}$$

which makes use of the linearity of $W$. The expectation operator is taken over the conditional distributions of $y_{+1}$ and $i_{+1}$.

In the CM, after the shocks are realized, the household's value functions are

$$W_u^h(a; s_{+1}) = b + l + \Delta(s_{+1}) + T(s_{+1}) + a + \max_{z_{+1} \geq 0}\left\{-\gamma z_{+1} + \beta U_u^h(z_{+1}; s_{+1})\right\},$$

and

$$W_j^h(\varepsilon, a; s_{+1}) = w_j(\varepsilon; s) + \Delta(s_{+1}) + T(s_{+1}) + a + \max_{z_{+1} \geq 0}\left\{-\gamma z_{+1} + \beta U_j^h(\varepsilon, z_{+1}; s_{+1})\right\},\ j \in \{e, i\}.$$

For firms, value functions in the LM are

$$U_u^f(0; s) = \alpha_f(s)\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{V_e^f(\varepsilon; s), V_i^f(\varepsilon; s)\right\}\ \mathrm{d}F(\varepsilon) + (1-\alpha_f)V_u^f(0; s)$$

and

$$U_j^f(\varepsilon; s) = (1-\delta)V_j^f(\varepsilon; s) + \delta V_u^f(0; s),\ j \in \{e, i\},$$

In the DM, the value functions are

$$V_e^f(\varepsilon; s) = \sigma_f(s)\left\{\eta[d_c(s) - c(q_c)] + (1-\eta)[d(s) - c(q(s))]\right\} + \mathbb{E}[W_e^f(\varepsilon, \varepsilon y, 0; s_{+1})]$$

for formal firms and

$$V_i^f(\varepsilon; s) = (1-\chi)\left\{\sigma_f(s)[d(s) - c(q(s))] + \mathbb{E}[W_i^f(\varepsilon, \varepsilon y, 0; s_{+1})]\right\} + \chi\beta U_i^f(\varepsilon; s_{+1})$$

for informal firms.

Finally, after all shocks are realized in the CM, firms' value functions are

$$W_e^f(\varepsilon, x, a; s_{+1}) = x - w_e(\varepsilon; s) - \tau_e + a + \beta U_e^f(\varepsilon; s_{+1}),$$

$$W_i^f(\varepsilon, x, a; s_{+1}) = x - w_i(\varepsilon; s) + a + \beta U_i^f(\varepsilon; s_{+1}),$$

and

$$W_u^f(s_{+1}) = \max\left\{0, -k + \beta U_u^f(s_{+1})\right\}.$$

In the LM, the surplus of a formal match is

$$S_e(\varepsilon; s) = V_e^f(\varepsilon; s) + V_e^h(\varepsilon, z; s) - V_u^h(\varepsilon, z; s)$$

and that of an informal match is

$$S_i(\varepsilon; s) = V_i^f(\varepsilon; s) + V_i^h(\varepsilon, z; s) - V_u^h(z; s).$$

The equilibrium wage $w_j(\varepsilon; s)$ satisfies the surplus sharing rule

$$V_j^f(\varepsilon; s) = (1-\omega) S_j(\varepsilon; s)$$

$$V_j^h(\varepsilon, z; s) - V_u^h(\varepsilon, z; s) = \omega S_j(\varepsilon; s)$$

for $j \in \{e, i\}$.

The bargaining solution in the DM is such that $d(z; s) = z$ and $q(z; s) = g^{-1}(z)$ where $z$ solves the CM maximization problem stated above and

$$g(q) = \varphi c(q) + (1-\varphi) v(q).$$

A Recursive Equilibrium is defined by functions $S_e$, $S_i$ that satisfy the Bellman equations

$$\begin{aligned} S_e(\varepsilon; s) = {} & R_e(\varepsilon; s) - \tau_e - b - l \\ & + \beta \mathbb{E}\left[(1-\delta) S_e(\varepsilon; s_{+1}) - \omega \alpha_h(s_{+1}) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{S_e(\varepsilon; s_{+1}), S_i(\varepsilon; s_{+1})\right\} \; \mathrm{d}F(\varepsilon)\right] \end{aligned}$$

and

$$\begin{aligned} S_i(\varepsilon; s) = {} & (1-\chi) R_i(\varepsilon; s) - b - l \\ & + \beta \mathbb{E}\left[(1-\delta) S_i(\varepsilon; s_{+1}) - \omega \alpha_h(s_{+1}) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{S_e(\varepsilon; s_{+1}), S_i(\varepsilon; s_{+1})\right\} \; \mathrm{d}F(\varepsilon)\right] \end{aligned}$$

and the free entry condition

$$k = \beta(1-\omega)\alpha_f(s) \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \max\left\{S_e(\varepsilon; s), S_i(\varepsilon; s)\right\} \; \mathrm{d}F(\varepsilon) \tag{B.2}$$

where $q$ solves

$$\frac{v'(q(s))}{g'(q(s))} = \frac{i}{\sigma_e(s)(1-\eta) + (1-\chi)\sigma_i(s)} + 1$$

and $\tilde{\varepsilon}$ solves

$$S_e(\tilde{\varepsilon}; s) = S_i(\tilde{\varepsilon}; s).$$

The above stochastic dynamical system is solved numerically as detailed in appendix C.

# Appendix C  Solution algorithm

The dynamic stochastic version of the model is solved globally using an approximated value function iteration algorithm:

1. Set grids for $\varepsilon$ and the aggregate state variables $s = \{y, i, u, n_i\}$.[32] For $y$ and $i$, use a discretization scheme to obtain state values and transition probabilities from their AR(1) processes.

2. Start with the non-stochastic steady state solution as an initial guess for $S_e$ and $S_i$.

3. Use a piece-wise linear interpolation scheme to interpolate $S_e$ and $S_i$ over the grids of $\varepsilon$ and the aggregate state $s$.

4. For each point $s$ in the aggregate state grid do the following:

   (a) Solve for $\tilde{\varepsilon}$ that satisfies

   $$\hat{S}_e(\tilde{\varepsilon}, s) - \hat{S}_i(\tilde{\varepsilon}, s) = 0$$

   where $\hat{S}_e$ and $\hat{S}_i$ are the interpolated value functions.

   (b) Using the obtained $\tilde{\varepsilon}$, calculate the integral

   $$\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \hat{S}_i(\varepsilon; s) \; \mathrm{d}F(\varepsilon) + \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} \hat{S}_e(\varepsilon; s) \; \mathrm{d}F(\varepsilon).$$

   over the distribution of productivities $F(\varepsilon)$.

[32] Since $u + n_e + n_i = 1$ it is enough to specify $u$ and $n_i$ as state variables.

(c) Use the obtained integral value to solve for $\theta$ from the free entry condition (B.2).

(d) Using $\theta$, $\tilde{\varepsilon}$ and the laws of motion (B.1), calculate $\{u', n_i', n_e'\}$.

(e) Given $i$, $\{u', n_i', n_e'\}$ and the resulting matching probabilities solve for $q$.

(f) For each point in the grid for $\varepsilon$ do the following:

i. Calculate $R_e(\varepsilon; s)$ and $R_i(\varepsilon; s)$ using $q$, the matching probabilities and $y$.

ii. For each point in the grids for $y'$ and $i'$ use to transition probability matrices to calculate the expectation terms in $S_e$ and $S_i$ as follows:

A. Solve for $\tilde{\varepsilon}'$ that satisfies

$$\hat{S}_e(\tilde{\varepsilon}, s') - \hat{S}_i(\tilde{\varepsilon}, s') = 0$$

B. Use it to calculate the integral

$$\int_{\underline{\varepsilon}}^{\tilde{\varepsilon}'} \hat{S}_i(\varepsilon; s') \, \mathrm{d}F(\varepsilon) + \int_{\tilde{\varepsilon}'}^{\overline{\varepsilon}} \hat{S}_e(\varepsilon; s') \, \mathrm{d}F(\varepsilon)$$

C. Use the integral to solve for $\theta'$ from the next period's free entry condition.

D. Use the obtained $\theta'$ and the integral to calculate the term inside the expectation for $S_e$ and $S_i$ and multiply by the corresponding conditional probability.

iii. Update the value functions at the corresponding grid points $S_e(\varepsilon; s)$ and $S_i(\varepsilon; s)$.

5. Check for convergence using the initial and updated value functions. If yes, stop. Otherwise, use the updated value functions in step 3 above and continue from there.

# Appendix D Analytical derivations

## D.1 Productivity threshold

The productivity threshold $\tilde{\varepsilon}$ satisfies

$$V_e^f(\tilde{\varepsilon}) = V_i^f(\tilde{\varepsilon}). \tag{D.1}$$

where

$$V_e^f(\varepsilon) = \frac{(1-\omega)\left(R_e(\varepsilon) - \tau_e - b\right) - \omega\theta k}{1 - \beta(1-\delta_e)},$$

and

$$V_i^f(\varepsilon) = \frac{(1-\omega)\left((1-\chi)R_i(\varepsilon) - b\right) - \omega\theta k}{1-\beta(1-\delta_i)}.$$

From equation (D.1) I get the expression

$$\tilde{\varepsilon} = \frac{(r_i - r_e)(b + \frac{\omega}{1-\omega}\theta k) + (r_e(1-\chi) - r_i(1-\eta))\sigma_f(g(q) - c(q)) - r_i(\sigma_f\eta(g(q_c) - c(q_c)) - \tau_e)}{[r_i - r_e(1-\chi)]y} \tag{D.2}$$

where $r_j \equiv 1 - \beta(1-\delta_j)$ for $j \in \{e, i\}$. Setting $\delta_e = \delta_i$, the above expression simplifies to

$$\tilde{\varepsilon} = \frac{\tau_e - \sigma_f\left[\eta(g(q_c) - c(q_c)) - (\eta - \chi)(g(q) - c(q))\right]}{\chi y}$$

which depicts the productivity threshold $\tilde{\varepsilon}$ as a function of $\sigma_f$, $q$, $q_c$ and the model parameters.

## D.2 Nash bargaining in the DM

Here I solve for the terms of trade in the DM using the generalized Nash bargaining solution. When a buyer is part of a pure monetary match, the bargaining problem can be formulated as follows:

$$\max_{q,d} \left[v(q) - d_c\right]^{\varphi} \left[d_c - c(q)\right]^{1-\varphi}$$

subject to:

$$d_c \leq z$$
$$c(q) \leq \varepsilon y$$

where $\varphi$ is the buyer's bargaining power. Since money is costly, households don't have an incentive to carry more money than they intend to spend in the DM market which makes the first constraint binding. In addition, I assume that the second constraint is never binding which allows us to write the problem as an unconstrained optimization problem. This results in the following first order condition:

$$z = \frac{\varphi v'(q)c(q) + (1-\varphi)v(q)c'(q)}{\varphi v'(q) + (1-\varphi)c'(q)} \equiv g(q)$$

The Nash bargaining solution is a pair $(q, d)$ that satisfies $q = g^{-1}(z)$ and $d_c = z$. Notice that $\partial q/\partial z = \partial g^{-1}(z)/\partial z = 1/g'(q) \geq 0$ meaning that more money holdings increases $q$.

Next, I solve for the terms of trade in credit formal matches:

$$\max_{q_c, d_c, \ell} [v(q_c) - d_c - \ell]^{\varphi} [d_c + \ell - c(q_c)]^{1-\varphi}$$

subject to:

$$\begin{aligned} d_c &\leq z \\ c(q_c) &\leq \varepsilon y \end{aligned}$$

The bargaining problem results in the following first order conditions:

$$\begin{aligned} z + \ell &= \frac{\varphi v'(q_c)c(q_c) + (1-\varphi)v(q_c)c'(q_c)}{\varphi v'(q_c) + (1-\varphi)c'(q_c)} \equiv g(q_c) \\ z + \ell &= (1-\varphi)v(q_c) + \varphi c(q_c) \end{aligned}$$

Combining both first order conditions we get the following optimality condition:

$$v'(q_c) = c'(q_c)$$

Hence the optimal solution is $q_c = q^*$ the efficient quantity which solves $v'(q) = c'(q)$. As a consequence, the Nash bargaining solution is a pair $(q_c, g(q_c))$ that satisfies $q_c = q^*$.

# Appendix E Proofs

## E.1 Proof of proposition 1

To ensure that the model equilibrium solution exhibits the single crossing property whereby $\exists!\ \tilde{\varepsilon} \in (\underline{\varepsilon}, \bar{\varepsilon})$ such that

- $V_e^f(\varepsilon) - V_i^f(\varepsilon) > 0, \forall\, \varepsilon \in (\tilde{\varepsilon}, \bar{\varepsilon}]$;
- $V_e^f(\varepsilon) - V_i^f(\varepsilon) < 0, \forall\, \varepsilon \in [\underline{\varepsilon}, \tilde{\varepsilon})$;
- $V_e^f(\tilde{\varepsilon}) - V_i^f(\tilde{\varepsilon}) = 0$ ,

it is sufficient to have the following conditions satisfied:

- $V_e^f(\varepsilon) - V_i^f(\varepsilon)$ strictly increasing in $\varepsilon$;
- $V_e^f(\bar{\varepsilon}) - V_i^f(\bar{\varepsilon}) > 0$;

- $V_e^f(\underline{\varepsilon}) - V_i^f(\underline{\varepsilon}) < 0$.

These conditions imply an additional set of restrictions on the model's parameters which I derive below.

The first condition implies

$$\frac{\partial\left(V_e^f(\varepsilon) - V_i^f(\varepsilon)\right)}{\partial\varepsilon} = \frac{(1-\omega)\chi y}{1-\beta(1-\delta)} > 0$$

which requires the additional restriction that $\chi > 0$. The second condition simplifies to

$$\chi\bar{\varepsilon}y + \sigma_f\eta[g(q_c) - c(q_c)] + \sigma_f(\chi - \eta)[g(q) - c(q)] > \tau_e.$$

Finally, the third condition simplifies to

$$\chi\underline{\varepsilon}y + \sigma_f\eta[g(q_c) - c(q_c)] + \sigma_f(\chi - \eta)[g(q) - c(q)] < \tau_e.$$

## E.2 Proof of lemma 2

For $\chi \in (0,1)$, we have

$$\tilde{\varepsilon} = \frac{\tau_e - \sigma_f\left[\eta(g(q_c) - c(q_c)) - (\eta - \chi)(g(q) - c(q))\right]}{\chi y}.$$

Taking the partial derivative with respect to $q$ yields

$$\frac{\partial\tilde{\varepsilon}}{\partial q} = \frac{\sigma_f(\eta - \chi)}{\chi y}(g'(q) - c'(qm)) > 0$$

for $\eta > \chi$ and $q < q^*$. Under the assumption $\eta > \chi$, we have:

$$\lim_{q\to 0}\tilde{\varepsilon} = \frac{\tau_e - \sigma_f\eta\left[g(q^*) - c(q^*)\right]}{\chi y} < \lim_{q\to q^*}\tilde{\varepsilon} = \frac{\tau_e - \sigma_f\chi\left[g(q^*) - c(q^*)\right]}{\chi y}.$$

Taking the partial derivative with respect to $\theta$ yields

$$\frac{\partial\tilde{\varepsilon}}{\partial\theta} = -\frac{\partial\sigma_f}{\partial u}\frac{\partial u}{\partial\theta}\left(\frac{\eta(g(q_c) - c(q_c)) - (\eta - \chi)(g(q) - c(q))}{\chi y}\right) > 0$$

where $\frac{\partial\sigma_f}{\partial u} > 0$ by virtue of the assumptions regarding the matching technology and $\frac{\partial u}{\partial\theta} < 0$ follows from the Beveridge curve. From $\lim_{\theta\to+\infty}\sigma_f \in (0,1)$ and $\lim_{\theta\to 0}\sigma_f = 1$, it is easy to see that

$$\lim_{\theta\to 0}\tilde{\varepsilon} = \frac{\tau_e - [\eta(g(q_c) - c(q_c)) - (\eta-\chi)(g(q) - c(q))]}{\chi y} < \lim_{\theta\to +\infty}\tilde{\varepsilon}.$$

## E.3 Proof of proposition 2

In the MD curve equation (23), totally differentiating with respect to $\theta$ yields

$$\frac{v''(q)g'(q) - v'(q)g''(q)}{[g'(q)]^2}\frac{dq}{d\theta} = i\left\{-\sigma_h\frac{d\rho(\tilde{\varepsilon})}{d\theta}\frac{\eta-\chi}{[1+\rho(\tilde{\varepsilon})]^2} - \frac{d\sigma_h}{d\theta}\left[1 - \frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right]\right\}$$
$$\left\{\sigma_h\left[1 - \frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right]\right\}^{-2}$$

Under the usual assumptions on the DM utility function ($v'(q) > 0, v''(q) < 0, \lim_{q\to 0} v'(q) = +\infty, \lim_{q\to\infty} v'(q) = 0$ ) and cost function ($c'(q) > 0, c''(q) \geq 0$) we have

$$\frac{v''(q)g'(q) - v'(q)g''(q)}{(g'(q))^2} = \frac{\varphi v''(q)c'(q) - \varphi v'(q)c''(q)}{(g'(q))^2} < 0.$$

Using this above and maintaining the assumptions $i > 0$, $\chi \in (0,1)$ and $\eta > \chi$ I get

$$\frac{dq}{d\theta} > 0.$$

We have $\lim_{\theta\to 0}\sigma_h = 0$ which implies $\lim_{\theta\to 0} q = 0$ since

$$\lim_{q\to 0}\frac{v'(q)}{g'(q)} = \lim_{q\to 0}\frac{v'(q)}{\varphi c'(q) + (1-\varphi)v'(q)} = +\infty > 0 \text{ for } \varphi \in (0,1).$$

We have $\lim_{\theta\to +\infty}\sigma_h \in (0,1)$

$$\lim_{q\to +\infty}\frac{v'(q)}{g'(q)} = \lim_{q\to +\infty}\frac{v'(q)}{\varphi c'(q) + (1-\varphi)v'(q)} = 0.$$

It follows that the solution to (23) exists and is unique.

## E.4 Proof of proposition 3

The JC condition (22) states that

$$k = \frac{(1-\omega)\alpha_f\left\{\int_{\tilde{\varepsilon}}^{\bar{\varepsilon}}[R_e(\varepsilon) - \tau_e]\ \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}}(1-\chi)R_i(\varepsilon)\ \mathrm{d}F(\varepsilon) - b\right\}}{1/\beta - 1 + \delta + \omega\alpha_h}.$$

Rearranging to get

$$(1/\beta - 1 + \delta + \omega\alpha_h)k = (1-\omega)\alpha_f \left\{ \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} [R_e(\varepsilon) - \tau_e] \; \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} (1-\chi) R_i(\varepsilon) \; \mathrm{d}F(\varepsilon) - b \right\}$$

and then totally differentiating with respect to $q$ using Leibniz rule yields

$$\omega \frac{d\alpha_h}{d\theta}\frac{d\theta}{dq}k = (1-\omega)\frac{d\alpha_f}{d\theta}\frac{d\theta}{dq} \left\{ \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} [R_e(\varepsilon) - \tau_e] \; \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} (1-\chi) R_i(\varepsilon) \; \mathrm{d}F(\varepsilon) - b \right\}$$
$$+ (1-\omega)\alpha_f \left\{ \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} \frac{dR_e(\varepsilon)}{dq} \; \mathrm{d}F(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} (1-\chi) \frac{dR_i(\varepsilon)}{dq} \; \mathrm{d}F(\varepsilon) \right\}$$

where I made use of the envelope property of $\tilde{\varepsilon}$ to simplify.[33] Applying the product rule I get

$$\frac{dR_e(\varepsilon)}{dq} = \frac{d\sigma_f}{d\theta}\frac{d\theta}{dq}\{\eta[g(q_c) - c(q_c)] + (1-\eta)[g(q) - c(q)]\} + \sigma_f(1-\eta)[g'(q) - c'(q)]$$

and

$$\frac{dR_i(\varepsilon)}{dq} = \frac{d\sigma_f}{d\theta}\frac{d\theta}{dq}[g(q) - c(q)] + \sigma_f[g'(q) - c'(q)].$$

From the definition of the matching function I get $\frac{\partial \alpha_f}{\partial \theta} < 0$ and $\frac{\partial \alpha_h}{\partial \theta} > 0$. By combining all what precedes, it is easy to see after some rearrangements that

$$\frac{d\theta}{dq} > 0$$

for $q < q^*$ and $\varphi < 1$. For $q = q^*$, $\bar{\theta}$ solves

$$k = \frac{(1-\omega)\alpha_f \left\{ \int_{\tilde{\varepsilon}}^{\overline{\varepsilon}} \varepsilon y + \sigma_f[g(q^*) - c(q^*)] - \tau_e \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon y + \sigma_f[g(q^*) - c(q^*)] \mathrm{d}F(\varepsilon) - b \right\}}{1/\beta - 1 + \delta + \omega\alpha_h}.$$

and we have

$$\left.\frac{d\theta}{dq}\right|_{q=q^*} = 0.$$

[33] Leibniz rule states that $\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)dt\right) = f(x, b(x))\frac{d}{dx}b(x) - f(x, a(x))\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)dt$ where $-\infty < a(x)$, $b(x) < +\infty$.

In what follows, I maintain the assumption that

$$(1/\beta - 1 + \delta)k < (1-\omega)\left\{ \int_{\varepsilon_e}^{\overline{\varepsilon}} \varepsilon y + [g(q^*) - c(q^*)] - \tau_e \; \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon y + [g(q^*) - c(q^*)] \; \mathrm{d}F(\varepsilon) - b \right\}$$

which guarantees the LM is at least active when $i \to 0$. Given that, if

$$(1/\beta - 1 + \delta)k > (1-\omega)\left\{ \int_{\varepsilon_e}^{\overline{\varepsilon}} \varepsilon y + \eta[g(q_c) - c(q_c)] - \tau_e \; \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon y \; \mathrm{d}F(\varepsilon) - b \right\}$$

then the JC curve passes through $(0, \underline{q})$ where $\underline{q}$ solves

$$(1/\beta - 1 + \delta)k = (1-\omega)\left\{ \int_{\varepsilon_e}^{\overline{\varepsilon}} \varepsilon y + \eta[g(q_c) - c(q_c)] + (1-\eta)[g(\underline{q}) - c(\underline{q})] - \tau_e \; \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon y + [g(\underline{q}) - c(\underline{q})] \; \mathrm{d}F(\varepsilon) - b \right\}.$$

If instead

$$(1/\beta - 1 + \delta)k \leq (1-\omega)\left\{ \int_{\varepsilon_e}^{\overline{\varepsilon}} \varepsilon y + \eta[g(q_c) - c(q_c)] - \tau_e \; \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon y \; \mathrm{d}F(\varepsilon) - b \right\}$$

then the JC curve passes through $(\underline{\theta}, 0)$ where $\underline{\theta}$ solves

$$(1/\beta - 1 + \delta + \omega\alpha_h)k = (1-\omega)\alpha_f\left\{ \int_{\varepsilon_e}^{\overline{\varepsilon}} \varepsilon y + \sigma_f\eta[g(q_c) - c(q_c)] - \tau_e \; \mathrm{d}F(\varepsilon) + (1-\chi) \int_{\varepsilon_i}^{\varepsilon_e} \varepsilon y \; \mathrm{d}F(\varepsilon) - b \right\}.$$

## E.5 Proof of Proposition 4

The result follows immediately from Propositions 2 and 3 and Lemma 2.

## E.6 Proof of Proposition 6

Since $i$ affects $\theta$ through the MD curve, I first show that $q$ is decreasing in $i$. For that, I totally differentiate equation (23) with respect to $i$:

$$\frac{v''(q)g'(q)-v'(q)g''(q)}{[g'(q)]^2}\frac{dq}{di}=\frac{1}{\sigma_h\left(1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)}-\frac{\left\{\frac{d\sigma_h}{dq}\left[1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right]+\sigma_f\frac{(\eta-\chi)}{[1+\rho(\tilde{\varepsilon})]^2}\frac{d\rho(\tilde{\varepsilon})}{d\tilde{\varepsilon}}\frac{d\tilde{\varepsilon}}{dq}\right\}i}{\left[\sigma_h\left(1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)\right]^2}\frac{dq}{di}.$$

Collecting the terms with $dq/di$ and rearranging yields:

$$\frac{dq}{di}=\frac{1}{\sigma_h\left(1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)}\left\{\frac{v''(q)g'(q)-v'(q)g''(q)}{[g'(q)]^2}+\frac{\left\{\frac{d\sigma_h}{du}\frac{du}{d\theta}\frac{d\theta}{dq}\left[1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right]+\sigma_f\frac{(\eta-\chi)}{[1+\rho(\tilde{\varepsilon})]^2}\frac{d\rho(\tilde{\varepsilon})}{d\tilde{\varepsilon}}\frac{d\tilde{\varepsilon}}{dq}\right\}i}{\left[\sigma_h\left(1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)\right]^2}\right\}^{-1}.$$

Given the assumptions on the utility and cost functions, it is easy to show that

$$\frac{v''(q)g'(q)-v'(q)g''(q)}{[g'(q)]^2}<0.$$

Evaluating $\frac{dq}{di}$ at $i=0$ yields

$$\left.\frac{dq}{di}\right|_{i\to 0}=\frac{1}{\sigma_h\left(1-\frac{\eta+\chi\rho(\tilde{\varepsilon})}{1+\rho(\tilde{\varepsilon})}\right)}\left\{\frac{v''(q)g'(q)-v'(q)g''(q)}{[g'(q)]^2}\right\}^{-1}<0.$$

Since I have shown in the proof of Proposition 3 (Appendix E.4) that $\frac{d\theta}{dq}>0$. It follows that $\frac{d\theta}{di}<0$ and $\frac{du}{di}>0$ at least in the neighborhood of the Friedman rule i.e. $i\to 0$.

To show that $\tilde{\varepsilon}$ is decreasing in $i$, I totally differentiate equation (18) with respect to $i$ to get

$$\frac{d\tilde{\varepsilon}}{di}=-\frac{\frac{d\sigma_f}{du}\frac{du}{d\theta}\frac{d\theta}{dq}\frac{dq}{di}\{\eta[g(q_c)-c(q_c)]-(\eta-\chi)[g(q)-c(q)]\}}{\chi}+\frac{\sigma_f\{(\eta-\chi)\frac{dq}{di}[\frac{dg(q)}{dq}-\frac{dc(q)}{dq}]\}}{\chi}<0$$

for $\eta \geq \chi > 0$ and $q < q^*$. From this it is straightforward to see that

$$\frac{dn_i}{di} = \frac{\frac{d\alpha_h}{di}F(\tilde{\varepsilon})\delta + \alpha_h \frac{dF(\tilde{\varepsilon})}{di}(\alpha_h + \delta)}{[\alpha_h + \delta]^2} < 0$$

which I obtained by totally differentiating equation (21) with respect to $i$.

Next, I show that the effect of $i$ on $n_e$ is ambiguous. Taking the total derivative of equation (20) with respect to $i$ yields

$$\frac{dn_e}{di} = \frac{\{\frac{d\alpha_h}{di}[1 - F(\tilde{\varepsilon})] - \alpha_h \frac{dF(\tilde{\varepsilon})}{di}\}(\alpha_h + \delta) - \{\alpha_h[1 - F(\tilde{\varepsilon})]\frac{d\alpha_h}{di}\}}{[\alpha_h + \delta]^2}$$

which simplifies to

$$\frac{dn_e}{di} = \frac{\frac{d\alpha_h}{d\theta}\frac{d\theta}{di}[1 - F(\tilde{\varepsilon})]\delta - \alpha_h \frac{dF(\tilde{\varepsilon})}{d\tilde{\varepsilon}}\frac{d\tilde{\varepsilon}}{di}(\alpha_h + \delta)}{[\alpha_h + \delta]^2}.$$

I have already established above that $\frac{d\theta}{di} < 0$ and $\frac{d\tilde{\varepsilon}}{di} < 0$. From the assumption on the matching function I get that $\frac{d\alpha_h}{d\theta} > 0$. Given that, the sign on $\frac{dn_e}{di}$ is ambiguous.

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