Terms of Trade Shocks and Labor Market Dynamics in Commodity-Exporting Economies

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Abstract

This paper investigates the empirical and theoretical relationship between commodity terms of trade shocks and the dynamics of the labor market in commodity-exporting economies. I build a two-sector small open economy RBC model with labor search frictions and a fixed commodity supply and calibrate it to Chilean data. In this model, commodity price shocks operate mainly through the income effect. The resulting movements in the real exchange rate affect the allocation of labor between the non-commodity tradable and non-tradable sectors. Compared to a frictionless economy, I show that labor search and matching frictions contribute to the dampening of the shock which helps explain the Terms of Trade disconnect discussed in the literature. In addition, persistence is much stronger and consumption is less volatile as vacancy creation provides the economy with an additional mechanism through which it can smooth external shocks. Finally, as is the case with productivity shocks in closed economies, I show numerically that the fundamental surplus fraction matters for the transmission of terms of trade shocks to unemployment in open economies.

Keywords: open economy; business cycle; labor market; search and matching; commodity.

JEL classification: F16, F41, F44.

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1 Introduction

The dominant view in the open economy macroeconomics literature holds that terms of trade shocks are a major driver of cyclical output fluctuations in emerging and developing countries. Mendoza (1995), Kose (2002) and Broda (2004), among others, report a contribution to the variability of GDP between 30% and 50%. Such results are obtained by estimating a stochastic process of the terms of trade and feeding it to an open economy real business cycle model. The resulting model-based conditional variance of output is then compared to the empirical unconditional variance to compute the share explained by terms of trade shocks. Another strand of the literature, mostly empirical, finds a much smaller impact of terms of trade shocks. For example, Schmitt-Grohé and Uribe (2018) estimate a country-specific structural vector auto-regression model using data from 38 developing countries and report that only 10% of the volatility of output can be attributed to terms of trade shocks with a substantial heterogeneity between countries. This discrepancy between theory and empirical evidence has been tentatively labeled the "terms of trade disconnect" (Schmitt-Grohé and Uribe, 2018).

In this paper I study the qualitative and quantitative impact of terms of trade shocks on the business cycle of a commodity-exporting economy. I contribute to the literature by focusing on two ideas: extending the standard two-sector small open economy model by adding labor market frictions and using commodity prices as a proxy for the terms of trade. The commodity output is modeled as a fixed endowment sold on international markets at an exogenous price subject to random shocks. The resulting revenue stream is transferred to households. As opposed to standard terms of trade shocks, commodity terms of trade shocks operate only through the income effect. A positive shock to the commodity price induces an increase in the price of non-tradable goods relative to tradable goods and hence an appreciation of the real exchange rate. Local firms react by shifting productive resources to the non-tradable sector. However, frictions in the labor market slow down the adjustment process as the reaction of employment to the shock is more muted compared to a frictionless economy. I focus the numerical analysis on Chile, an emerging economy with large commodity exports. I use a calibrated version of the model to assess the contribution of commodity terms of trade shocks to Chilean output and labor market fluctuations. In order to better analyze the role of labor search frictions in the adjustment process, I compare the model with labor search frictions to the standard real business cycle (RBC) model. I show numerically that the aggregate reaction of output and employment is dampened in the presence of labor search frictions which can help explain the terms of trade disconnect (Schmitt-Grohé and Uribe, 2018). Fluctuations in the real exchange rate resulting from terms of trade shocks generate a significant sectoral reallocation of labor under labor search frictions. This result is particularly amplified when capital is included in the model. The presence of capital along labor search frictions is also essential to replicate the negative reaction of unemployment to terms of trade shock. In addition, the resulting persistence is much stronger and consumption is less volatile as the creation of vacancies offers the economy an additional tool through which to smooth external shocks. However, the model generates negative cross-sectoral correlation of employment which is at odds with the data. Finally, I show numerically that the fundamental surplus fraction matters of the transmission of terms of trade shocks to unemployment as is the case with productivity shocks in a closed economy (Ljungqvist and Sargent, 2017).

The first idea pursued in this paper is that the presence of real frictions in the labor market can provide a possible buffer to terms of trade shocks in emerging economies. These frictions would operate as a labor adjustment cost both within and across sectors. Without labor adjustment costs, real marginal costs are equal to unit labor costs, i.e. the ratio of real wages to labor productivity. In addition, the slower sectoral labor reallocation between tradable and non-tradable sectors can theoretically contribute to a dampening of the terms of trade shocks and thus reduce the volatility of output compared to a frictionless economy. As opposed to more reduced form labor adjustment costs which are determined at the level of the firm, labor search frictions are determined endogenously based on aggregate labor market conditions (Krause et al., 2008).

The second idea of this paper is to use commodity terms of trade instead of the more general terms of trade to understand the impact of external shocks on developing and emerging economies. First, commodity terms of trade offer a better alternative to the extent that they can pass the exogeneity test more successfully (Schmitt-Grohé and Uribe, 2016; Fernández et al., 2017). Commodity terms of trade are defined as the price of a country's commodity exports in terms of its commodity imports (Aslam et al., 2016). Commodities are often traded in liquid international markets with standardized contracts. Apart probably from geopolitical risks linked to supply disruptions, macroeconomic conditions in commodity-exporting economies often have little impact on market prices even when these countries hold a significant market share. This is especially true in the case of mature commodities where supply is inelastic in the short and medium run and price fluctuations are mostly driven by factors affecting global demand. Second, from a more theoretical perspective using commodity terms of trade shocks to an exogenous commodity endowment allows to abstract from the direct substitution effect of terms of trade shocks and focus on the income effect.

This paper contributes also to the debate about unemployment volatility in response to productivity shocks (Shimer, 2005). I show numerically that a smaller fundamental surplus fraction, i.e. the fraction of a job's output allocated to recruitment activities, results in a higher volatility of unemployment as a reaction to terms of trade shocks in line with the results in the case of productivity shocks (Ljungqvist and Sargent, 2017). This result is particularly important for commodity-exporting economies with regard to stabilization policies.

The paper is organized as follows. Section 2 discusses briefly the related literature. Section 3 introduced some stylized facts on the relationship between commodity prices and labor market dynamics. Section 4 presents the baseline model and discusses some of its theoretical properties. I present in section 6 the model's calibration and discuss the numerical findings. Section 7 concludes.

2 Related literature

Search and matching frictions in the tradition of Diamond-Mortensen-Pissarides (DMP) have been studied extensively as a plausible alternative to the frictionless Walrasian model of the labor market in closed economies. This way of thinking about the labor market permits the modeling of unemployment in equilibrium. The use of labor search and matching frictions in different dynamic general equilibrium models results in an improved performance in matching the data over several dimensions (Yashiv, 2007; Rogerson and Shimer, 2011). However, there remains a strong debate both on the way wage determination should be modeled and on the calibration which together affect the ability of the DMP model to account for some stylized facts at the business cycle frequency (Shimer, 2005; Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Hall, 2017; Haefke and Reiter, 2017).

In the DMP framework, job matches require a resource consuming search process and thus result in a situation of bilateral monopoly which generates a positive surplus for the matched firm and worker. In a frictionless model, the marginal product of labor (MPL), the marginal rate of substitution between consumption and leisure (MRS), which accounts for the disutility of labor in terms of consumption units, and the wage are all equal. In a model with labor search frictions, being matched allows not only for production but also for economies on future search costs. This implies the existence of gains from trade in the form of a positive surplus of the MPL on the MRS that can be shared between the worker and the firm. The way the wage is set decides how these gains from trade are shared. Workers are willing to accept any wage as long as it is equal to or above their MRS while firms are willing to accept any wage that is equal to or below their MPL. As long as the wage is inside that range it is an equilibrium wage. As a consequence of this indeterminacy, the wage setting mechanism plays an important role in the determination of labor market dynamics.

The seminal work of Shimer (2005) has started a decade-long debate on the transmission of productivity shocks to unemployment in models with labor search frictions (Hall, 2005; Mortensen and Nagypal, 2007; Shimer, 2010; Hall and Milgrom, 2008; Hagedorn and Manovskii, 2008; Rogerson and Shimer, 2011). Shimer (2005) argues that the volatility of the match surplus resulting form productivity shocks is almost completely transmitted to wages as a result of the Nash bargaining solution. For plausible parameterizations, this high wage volatility implies tiny variations in unemployment which is at odds with empirical evidence. Other authors tried to solve this puzzle either by making wages more rigid, by adding more costs or by arguing for a different calibration. This debate was arguably ended by Ljungqvist and Sargent (2017) who provide an elegant explanation based on the fundamental surplus fraction, i.e. an upper bound on the share of a job's output that the market can allocate to vacancy creation. The authors show that a smaller fundamental surplus fraction results in a higher elasticity of market tightness to productivity shocks. Hall (2017) show that this result extends to other types of shocks in closed economies. However, this issue has not been fully explored in the case of terms of trade shocks to small open economies. I contribute to this issue by showing numerically that the fundamental surplus fraction matters of the transmission of terms of trade shocks to unemployment.

The first paper to study endogenous sectoral labor reallocation in the context of labor search frictions is Chang (2011). The author introduces labor reallocation costs to a closed-economy two-sector DMP model and studies the reaction to aggregate and sectoral shocks. As opposed to Chang (2011), the environment I consider here does not require the use of intra-firm wage bargaining for a two-sector labor market to exist in the steady state when productivity between the two sectors is different.

Few authors studied the impact of introducing labor search frictions into small open economy (SOE) business cycle models. The first are probably Feve and Langot (1996) who compare the ability of three models to match the French business cycle statistics: a standard RBC model, its extension to an open economy setting and an open economy version with labor search frictions and wage bargaining. After estimating the models' parameters, they find that the third version presents the best performance. They explain this result by the ability of the model with search frictions to reproduce labor market stylized facts and in particular capture the persistent unemployment levels observed in European countries. In addition, the authors report that European countries, as opposed to the US, exhibit a lower volatility of employment compared to labor productivity which is in line with the predictions of the standard DMP model with Nash bargaining.

In a similar setting, Boz et al. (2015) examine the interaction between productivity shocks and interest rate shocks on business cycle fluctuations in emerging countries. The higher interest rate accompanying a negative productivity shock induces firms to discount more heavily future profits and hence limit their vacancy creation. The same shocks lead to a decrease in workers' outside option due to the lower prospects of job finding as well as lower expected job surplus. This fall in the outside option of workers feeds into lower wages through the bargaining process. At the same time, higher employment uncertainty leads to an increase in precautionary saving which further reduces consumption and depresses the economy.

Bodenstein et al. (2018) study the relationship between commodity price shocks and unemployment in advanced commodity exporting economies. They estimate an SVAR model of the Norwegian economy and then use the resulting impulse response functions to calibrate a small open economy RBC model with labor search frictions. As in my paper, their main finding is that commodity prices affect labor market conditions through the real exchange rate. An increase in commodity prices works as a wealth transfer to households which rises goods consumption. This prompts non-commodity firms to post additional job vacancies which in turn reduces unemployment. The authors find also that a substantial limit on international risk sharing is needed in order to match the data. Compared to Bodenstein et al. (2018), I compare the performance of the labor search model with a frictionless real business cycle model in order to disentangle the contribution of search and matching frictions.

My paper is close to Medina and Naudon (2011) who look at the impact of both mining and non-mining terms of trade shocks on the Chilean labor market. First, they estimate an SVAR model of the Chilean economy to evaluate the empirical relationship between terms of trade shocks and labor market variables such as unemployment, the job finding rate and sectoral employment. In a second step, they build a multi-sector model with importable, exportable and non-tradable goods sectors as well as a commodity sector. They find out that the presence of a high level of wage regidity can help the model reproduce a sharp fall in unemployment after a mining terms of trade shock. However, their model produces far more sectoral reallocation of labor than observed in the data. Compared to their work, I focus only on mining terms of trade shocks and adopt a simpler tradables/non-tradables structure in order to disentangle the different channels through which commodity price shocks operate. I also compare the numerical performance of the models with labor search frictions to the standard SOE RBC model.

As an alternative to labor market frictions, Shousha (2016) studies the effect of financial frictions on the reaction of output to commodity price shocks. As he notes, output growth in advanced commodity exporters doesn't react to the commodity cycle as much as in less advanced countries. One reason behind this could be the development of the financial sector and the resulting degree of financial frictions in the economy. Using a panel SVAR analysis on the two groups of countries, the author shows that commodity price shocks have a stronger effect on both the real activity and financial conditions in emerging economies. An interesting result is that the inclusion of commodity price shocks in the analysis renders the contribution of interest rate shocks to output fluctuations almost negligible. This result stands in contrast with the findings of Neumeyer and Perri (2005) and Uribe and Yue (2006) with the caveat that these papers didn't focus on commodity-exporting economies. One way of reconciling these results would be that commodity price shocks are the major driver of country risk premia facing commodity-exporting emerging countries as opposed to non-commodity exporters where interest rate can be driven by non-commodity related factors. This point is further explored by Shousha (2016) in an SOE RBC model with financial frictions. The model shows that the reaction of the economy to commodity terms of trade shocks can be amplified when coupled with co-movements in interest rates. The amplification mechanism affects the economy through a working capital channel. The more severe are the financial constraints facing firms the stronger the amplifications of the commodity price shock. These results are confirmed in the context of Argentina by Drechsel and Tenreyro (2018) who allow various shocks to compete using a Bayesian estimated SOE RBC model and find that, combining both the real exchange rate and the interest rate channels, commodity price shocks can explain 38% of output fluctuations.

It is worth mentioning that the open economy literature makes an extensive use of the assumption of the exogeneity of the terms of trade. This assumption doesn't seem implausible in the case of small open economies. It is somewhat relaxed by the semi-open economy literature (Gali and Monacelli, 2005) where the terms of trade are endogenous.

3 Commodity prices and the business cycle

Commodities play an important role in emerging and developing market economies. Using data on 189 countries between 1960 and 2013, Rodriguez et al. (2015) report that the median share of commodities in the exports of emerging economies stands at 25.7%, more than double that of advanced economies which stands at 11.2%. This explains the attention given by the open economy literature to commodity price shocks as a driver of emerging economies' business cycle. In this section, I summarize the evidence produced in the literature on the relationship between commodity terms of trade and the business cycle of commodity-exporting countries.

The standard channel explored by most of the literature is the wealth channel. An increase in commodity prices means the existing levels of commodity production generate higher revenues. This increases the wealth of households which translates into higher consumption and saving. The increase in non-commodity output is subdued since part of the increase in income is used in the consumption of imports. This means that the domestic supply reaction to the commodity shock occurs mainly in the non-tradable sector. One implication of this is a shifting of production factors away from the non-commodity tradables. This theoretical result is known as the Dutch disease. This phenomenon has been studied by an extensive theoretical and empirical literature which was mostly concerned by the long run impact of an increase in commodity export revenues on the allocation of production factors in the economy. The seminal work of Corden and Neary (1982) distinguishes between the spending effect and the resource movement effect. The former is what I discussed above as the wealth effect. The latter is a more direct effect working through an increased investment in the commodity sector and the ensuing higher demand from commodity producers for inputs from the rest of the economy (e.g. construction, transport, services).

As shown by Céspedes and Velasco (2012), the reaction of commodity-exporting economies to commodity price shocks varies largely between countries and depends both on the structural characteristic of the economy and on the policy framework in place. In countries where a significant share of government income is tied to commodity exports, fiscal policy can play an important role in the transmission of commodity price shocks to the extent that the volatility of commodity prices can transmit to government income. If public expenditures react more than proportionally to changes in public revenues this may result in a pro-cyclical fiscal balance (Céspedes and Velasco, 2014).

Most of the empirical evidence points to substantial differences in labor market fluctuations over the business cycle between developing and developed countries (Neumeyer and Perri, 2005; Li, 2011; Boz et al., 2015). Li (2011) presents evidence that real wages are positively correlated with output in developing countries in contrast to advanced countries where no systematic pattern is detected. In addition, the ratio

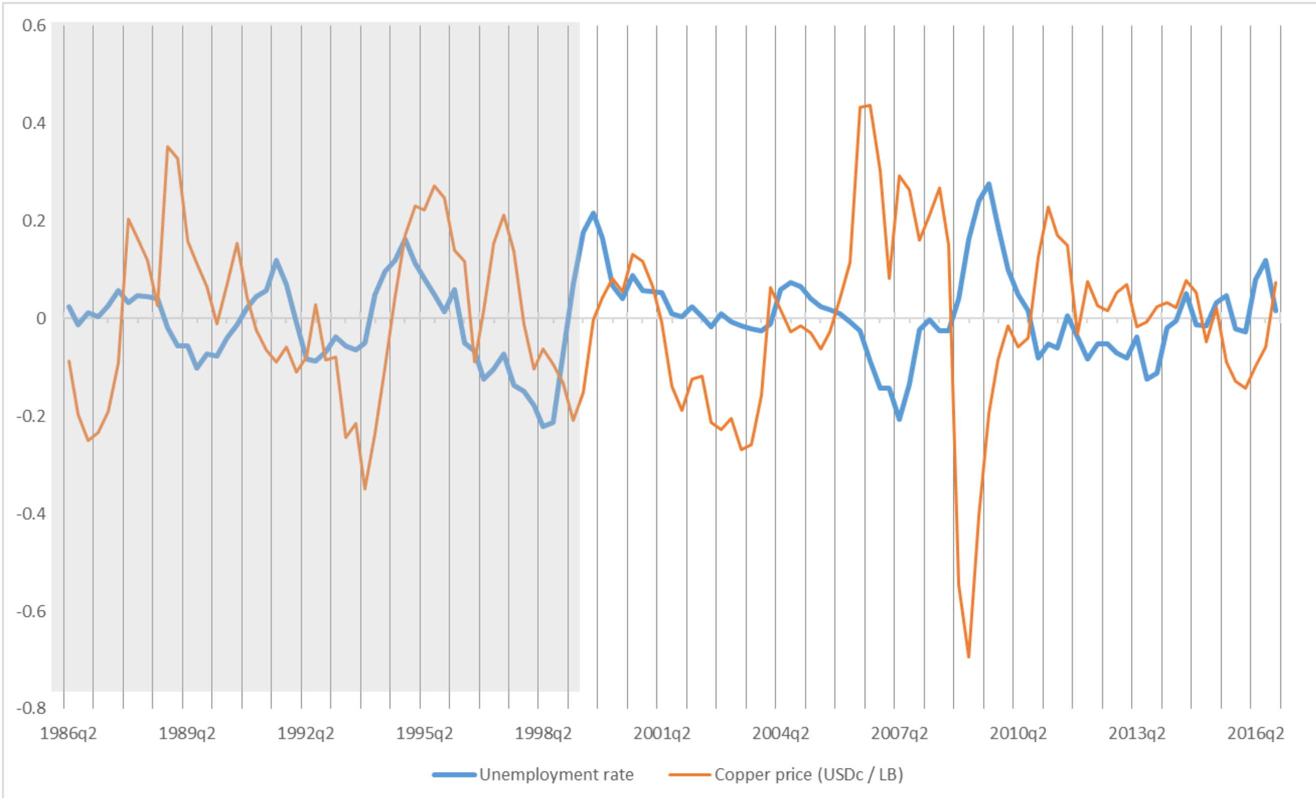


Figure 1: Unemployment rate in Chile and the Copper price (HP filtered)

of the volatility of real wages to GDP is twice as high in developing countries as opposed to advanced economies. Another important regularity that has strong theoretical implications was pointed out by Boz et al. (2015). The authors show evidence that fluctuations in prices are large while fluctuations in quantities are less pronounced compared to advanced economies. This implies that the Shimer (2005) critic to the DMP framework doesn't necessarily apply to developing economies. For example, real wages are twice as volatile in their sample as unemployment. One possible explanation is the absence of strong labor institutions that might render wages more rigid as in advanced countries.

To shed some light on how labor markets react to commodity price shocks I provide some evidence from the Chilean economy. Copper accounts for around 50% of exports and 10% of real GDP in Chile. Chile adopted a flexible exchange rate regime starting from 1999. Since then Copper prices went through two cycles. Table 1 presents some business cycle statistics over two separate periods: from 1986 to 1999 before the floating of the nominal exchange rate and then from 1999 to 2016 after the floating. The second period covers the two recent commodity cycles.

The Chilean business cycle exhibits roughly the same stylized facts in developing and emerging

Table 1: Business cycle statistics for Chile

HP filtered data	**1986Q2-1999Q3**	**1999Q4-2016Q4**
Copper price (log)		
Standard Deviation	0.1613	0.1963
Rel. Standard Deviation	8.0000	11.800
Real GDP (log)		
Standard Deviation	0.0203	0.0167
Corr. with Copper price	0.0859	0.6449
Private Consumption (log)		
Standard Deviation	N/A	0.0218
Rel. Standard Deviation	N/A	1.3000
Corr. with Output	N/A	0.9288
Corr. with Copper price	N/A	0.7388
Investment (log)		
Standard Deviation	0.0759	0.0603
Rel. Standard Deviation	3.7000	3.6000
Corr. with Output	0.7457	0.7963
Corr. with Copper price	0.1982	0.3943
Unemployment		
Standard Deviation	0.0086	0.0073
Rel. Standard Deviation	0.4000	0.4000
Corr. with Output	-0.7217	-0.6779
Corr. with Copper price	0.0327	-0.5547
Employment in Tradables		
Standard Deviation	0.0025	0.0031
Rel. Standard Deviation	0.1000	0.2000
Corr. with Output	0.4739	0.4060
Corr. with Copper price	0.1501	0.3098
Employment in Non-Tradables		
Standard Deviation	0.0046	0.0053
Rel. Standard Deviation	0.2000	0.3000
Corr. with Output	0.3825	0.5886
Corr. with Copper price	-0.2323	0.4110

Source: Haver Analytics, author's calculations

economies reported in the literature (Neumeyer and Perri, 2005; Li, 2011; Durdu, 2013; Boz et al., 2015). Consumption is more volatile than output reflecting a lower ability of agents to smooth consumption perhaps as a result of a less developed financial sector. Investment is much more volatile than output or consumption. After the floating of the exchange rate, all economic indicators were strongly correlated with Copper prices as opposed to the period with rigid nominal exchange rate where the relationship was relatively weak. For instance, the cyclical component of unemployment went from being completely disconnected from Copper price fluctuations to having a strong negative correlation. This change can clearly be seen in figure 1 which plots cyclical fluctuations of unemployment and the Copper price. During the second period, both employment in the non-tradables and the non-mining tradables are positively correlated with Copper price fluctuations. This was not the case during the fixed exchange rate period when employment in the non-tradable sector was negatively correlated with the Copper price.

4 Baseline model

In order to better understand how commodity terms of trade shocks affect the economy in general and labor market dynamics in particular, I consider a small open economy business cycle model with search frictions in the labor market as in Andolfatto (1996), Merz (1995) and Shimer (2010), and a commodity sector. In the main text I present a version of the model without capital. A version with capital is presented in the appendix.

4.1 Environment

Time is discrete and continues forever. The economy is populated by a representative household with the usual preferences for consumption and leisure. The household is composed of a unit measure of ex-ante identical members. Household members live infinitely and discount the future with factor β. The representative household maximizes the average utility of his members with equal weights. First used by Lucas (1980), the large household structure allows for full risk sharing between the household's members which avoids the complications arising from heterogeneity (Shimer, 2010; Blanchard and Galí, 2010).

Three goods exist in this economy: a non-tradable good y^N, consumed only locally and for which the price is determined on the local market; a non-commodity tradable good y^T consumed both locally and abroad and for which the price is determined on the international market and taken as given by the local economy; and a commodity tradable good y^C, destined exclusively for export at an internationally determined price. Perfectly competitive domestic firms operate in this economy in order to produce the non-tradable and the non-commodity tradable goods. Firms use labor in order to produce both types of goods. The excess (deficit) of production over domestic absorption of the non-commodity tradable good is exported (imported) to (from) the rest of the world. The domestic economy is endowed each period with a exogenously fixed supply of the commodity good.

4.2 Labor market

Members of the household face stochastic job opportunities. In particular, an individual can be either employed or unemployed and searching for a job in one of the economy's two sectors. At the household level, we define n_t as the fraction of employed members and $u_t = 1 - n_t$ as the fraction of unemployed members. I ignore all considerations related to labor participation by assuming the labor force to be constant and normalized to 1.

Firms in sector $j \in \{T, N\}$ divide their workforce n_t^j into recruiting v_t^j and production $1 - v_t^j$. The recruiters are tasked with filling job vacancies by attracting new workers to the firm. I rule out on the job search by assuming that recruiters attract only unemployed workers. The number of matches a recruiter is able to attract in each sector of the economy is governed by a constant returns to scale matching technology

$$m(v_t^j n_t^j, u_t^j) = A(v_t^j n_t^j)^\gamma (u_t^j)^{(1-\gamma)}$$

where A is the matching efficiency parameter and γ is an elasticity parameter. I define θ^j, the labor market tightness in sector $j \in \{T, N\}$, as the ratio of recruiters $v_t^j n_t^j$ to job seekers u_t^j

$$\theta^j = \frac{v^j n^j}{u^j} \tag{1}$$

and use it to rewrite the matching function as

$$m(\theta_t^j, u_t^j) = A(\theta_t^j)^\gamma u_t^j.$$

Firms in sector $j \in \{T, N\}$ fill their vacancies with probability $q_t^j(\theta_t^j) \equiv \frac{m(\theta_t^j, u_t^j)}{v_t^j n_t^j} = A(\theta_t^j)^{\gamma-1}$ while workers searching for jobs in sector j find one with probability $\pi_t^j(\theta_t^j) \equiv \frac{m(\theta_t^j, u_t^j)}{u_t^j} = A(\theta_t^j)^\gamma$. Existing matches in sector j are destroyed at an exogenous rate ρ^j. As a result, employment in sector j evolves according to the law of motion

$$n_{t+1}^j = (1 - \rho^j) n_t^j + \pi_t^j u_t^j. \tag{2}$$

Given the values of n^j and u^j in each sector, the definition of the labor force

$$u_t^T + u_t^N + n_t^T + n_t^N = 1 \tag{3}$$

has to hold in every period.

4.3 Households

In addition to the standard consumption-saving problem, households maximize their members' lifetime utility by choosing the consumption level of their employed and unemployed members, the composition of the average consumption basket and the share of unemployed members searching for work in each sector.

I assume the period utility of a household member takes the form

$$\frac{(c_t^e - b)^{1-\sigma} - 1}{1-\sigma}$$

if the member is employed and consumes c_t^e and

$$\frac{(c_t^u)^{1-\sigma} - 1}{1-\sigma}$$

if the member is unemployed and consumes c_t^u. Parameter $b > 0$ measures the disutility of labor while σ determines both risk-aversion and the inter-temporal elasticity of substitution. This utility specification is an indivisible labor version of the standard GHH preferences (Greenwood, Hercowitz, and Huffman, 1988).[1] The use of GHH preferences is useful as it shuts down wealth effects on labor supply by making the marginal rate of substitution (MRS) between consumption and leisure independent of the level of consumption. The reason I use these preferences is that if the wealth effect is active, persistent positive productivity or terms of trade shocks can result in a decrease in employment. This goes against the empirical evidence on the procyclicality of employment.

The representative household chooses consumption levels c_t^e and c_t^u that maximizes the sum of the utilities of all its members

$$\frac{(c_t^e - b)^{1-\sigma} - 1}{1-\sigma} n_t + \frac{(c_t^u)^{1-\sigma} - 1}{1-\sigma}(1 - n_t) \tag{4}$$

subject to

$$c_t = c_t^e n_t + c_t^u (1 - n_t)$$

where c_t is the total (average) consumption basket of tradable and non-tradable goods consumed by the

[1] See Uribe and Schmitt-Grohé (2017) for a discussion of GHH preferences in the context of the small open economy real business cycle model.

household and n_t is the employed fraction of household members. The optimal solution yields

$$c_t^e = c_t^u + b$$

meaning that employment risk sharing within the household requires compensation of employed members for their disutility of labor through a higher consumption relative to unemployed members. Replacing the above equation in the household's total consumption basket we get

$$c_t^u = c_t - bn_t \tag{5}$$

and

$$c_t^e = c_t + b(1 - n_t). \tag{6}$$

Plugging back (5) and (6) in the household's objective function (4) we get

$$U(c_t, n_t) = \frac{(c_t - bn_t)^{1-\sigma} - 1}{1 - \sigma}$$

where the household behaves as if it has a utility function defined over average consumption c_t and labor supply n_t.[2] In particular, we have

$$\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = \frac{-b(c_t - bn_t)^{-\sigma}}{(c_t - bn_t)^{-\sigma}} = -b$$

where the MRS at the household's level is constant and hence independent of the level of consumption. As explained above, this allows us to abstract from wealth considerations when analyzing the effect of terms of trade shocks on unemployment. The reason the MRS is constant in this setting has to do with the non-convexity resulting from the indivisibility of labor supply at the household member's level which takes only the values 0 or 1. Including an intensive margin would allow a variable MRS if labor disutility is a non-linear (strictly convex) function of the number of hours worked. As most of the variation in employment at the business cycle frequency occurs along the extensive margin, I can safely abstract from the intensive margin in the model with search frictions.

The consumption basket c_t consists of the consumption of tradables c_t^T, and non-tradables c_t^N. More

[2] See Shimer (2010) for a similar derivation using balanced growth path preferences.

formally, c_t is defined as the constant elasticity of substitution (CES) index

$$c_t = \left[\varphi^{\frac{1}{\omega}}(c_t^N)^{\frac{\omega-1}{\omega}} + (1-\varphi)^{\frac{1}{\omega}}(c_t^T)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

where $\varphi \in (0,1)$ is the share of non-tradable goods in the consumption basket and ω is the intratemporal elasticity of substitution between tradable and non-tradable goods.

Taking the tradable good as the numeraire, the optimal allocation of household's expenditures between tradable and non-tradable goods subject to

$$p_t c_t = p_t^N c_t^N + c_t^T$$

implies the following demand function

$$\frac{c_t^T}{c_t^N} = \frac{1-\varphi}{\varphi}(p_t^N)^{\omega} \tag{7}$$

where p_N is the price of non-tradable goods relative to tradable goods. After some tedious algebra, one can obtain the price index for aggregate consumption in terms of the tradable good

$$p_t = \left[\varphi(p_t^N)^{1-\omega} + (1-\varphi)\right]^{\frac{1}{1-\omega}}. \tag{8}$$

Assuming the law of one price holds for tradable goods, the inverse of p_t corresponds to the real exchange rate, i.e. the relative price of one unit of the foreign consumption basket in terms of the domestic one. To see that start from

$$RER_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate and P_t and P_t^* are the domestic and foreign nominal consumption price indices in terms of the domestic and foreign currency respectively. Using the law of one price for tradable goods we have $P_t^T = \mathcal{E}_t P_t^{T*}$ which we can use in the above equation after dividing both numerator and denominator by P_t^T to get

$$RER_t = \frac{P_t^*/P_t^{T*}}{P_t/P_t^T}.$$

The numerator depicts the foreign relative price index of the consumption basket in terms of the tradable good which I normalize to 1. The denominator depicts the domestic relative price index of the consumption basket in terms of the tradable good which corresponds to p_t. The real exchange rate can be written as

$$RER_t = \frac{1}{p_t} = \frac{1}{\left[\varphi(p_t^N)^{1-\omega} + (1-\varphi)\right]^{\frac{1}{1-\omega}}}$$

meaning that the domestic economy becomes expensive relative to the rest of the world if and only if non-tradable goods become expensive relative to tradable goods, i.e. an increase in p^N.

The household starts the period with the following state variables $\mathcal{S}_t = \{a_t, n_t^T, n_t^N\}$ where a_t are its international asset holdings and n_t^j is the measure of its employed members in sector j. The household maximizes the total utility of its members by choosing how much to spend in consumption and save in the internationally traded asset a_t and the share of its unemployed members to send in search for vacancies in each sector. This problem can be stated in the following recursive formulation:

$$V(\mathcal{S}_t) = \max_{c_t, u_t^T, u_t^N, a_{t+1}} \{U(c_t, n_t) + \beta \mathbf{E}[V(\mathcal{S}_{t+1})]\}$$

subject to the following constraints:

$$\begin{aligned}
p_t c_t + a_{t+1} &= w_t^N n_t^N + w_t^T n_t^T + (1+r_t)a_t + \Pi_t + \Pi_t^C; \\
1 &= u_t + n_t; \\
u_t &= u_t^N + u_t^T; \\
n_t &= n_t^N + n_t^T; \\
n_{t+1}^j &= (1-\rho^j)n_t^j + \pi_t^j u_t^j \quad \text{for } j \in \{N, T\},
\end{aligned}$$

where Π_t and Π_t^C are profits transfered from non-commodity firms and the revenues generated from commodity exports respectively, and w_t^j is the real wage earned by household members working in sector $j \in \{N, T\}$.

Solving for the first order and envelope conditions related to the consumption-saving decision yields the usual Euler equation

$$\frac{U_c(c_t, n_t)}{p_t} = \beta \mathbf{E}\left[(1+r_{t+1})\frac{U_c(c_{t+1}, n_{t+1})}{p_{t+1}}\right] \tag{9}$$

while the first order condition for u_t^j yields

$$\pi_t^T \beta \mathbf{E}[V_{n^T}(\mathcal{S}_{t+1})] = \pi_t^N \beta \mathbf{E}[V_{n^N}(\mathcal{S}_{t+1})]. \tag{10}$$

The latter states that the optimal level of household members searching for work in each sector equalizes next period's marginal value of landing a job in each sector weighted by the corresponding probability of job finding. In equilibrium, this implies that the household will be indifferent between the two sectors given their respective wage and labor market tightness. Notice that given n_t, choosing either u_t^N or u_t^T will determine the other.

Finally, I solve for the envelope condition with respect to n_t^j

$$V_{n^j}(\mathcal{S}_t) = U_c(c_t, n_t)\frac{w_t^j}{p_t} + U_n(c_t, n_t) + (1 - \rho^j - \pi_t^j)\beta \mathbf{E}[V_{n^j}(\mathcal{S}_{t+1})] \tag{11}$$

which states that the marginal value to the household of an additional member finding a job in sector j is equal to the sum of the marginal utility of consumption resulting from the received wage, the ensuing disutility of working and the discounted continuation value weighted by the probability of keeping the job next period. This equation, defined for each sector respectively, along with the indifference equation (10) determine the equilibrium allocation of job seekers given the current level of unemployed members of the household. Notice that u_t^j, as opposed to u_t, is a jump variable as the household can fully adjust the sectoral allocation of job seekers following changes in the economic environment.

4.4 Firms

A representative firm operating in sector $j \in \{T, N\}$ employs n_t^j workers. It assigns a fraction v_t^j of its workforce to recruiting activities and the remaining fraction $1 - v_t^j$ to production. The firm uses the labor of production workers to produce good j using the production function

$$y_t^j = z_t^j n_t^j (1 - v_t^j)$$

where z_t^j is total factor (labor) productivity in sector j. Firms use recruiters v_t^j to fill vacancies. Each recruiter attracts q_t^j workers while an exogenously determined fraction ρ^j of workers quits the firm every period.

The representative firm in sector j starts the period with the following state variables $\mathcal{S}_t^j = \{z_t^j, n_t^j\}$ and chooses recruiting and production efforts in order to maximize the present value of its profits

$$J^j(\mathcal{S}_t^j) = \max_{v_t^j \in [0,1]} p_t^j y_t^j - w_t^j n_t^j + \tilde{\beta}\mathbf{E}[J^j(\mathcal{S}_{t+1}^j)]$$

subject to:

$$\begin{aligned} y_t^j &= z_t^j n_t^j (1 - v_t^j) \\ n_{t+1}^j &= n_t^j (v_t^j q_t^j + 1 - \rho_j) \\ \log z_{t+1} &= (1 - \rho_z) \log \bar{z} + \rho_z \log z_t + \sigma_z \varepsilon_{t+1} \end{aligned} \tag{12}$$

where

$$\tilde{\beta} = \mathbf{E}\left[\frac{1}{1 + r_{t+1}}\right] = \beta \mathbf{E}\left[\frac{U_c(c_{t+1}, n_{t+1})/p_{t+1}}{U_c(c_t, n_t)/p_t}\right]$$

is the stochastic discount factor derived from households' optimality condition (9), $\rho_z \in (0, 1)$ is the persistence of the productivity process z_t, σ_z is its conditional volatility and ε_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock.

Assuming an interior solution, the first order condition with respect to the share of recruiters in the workforce v_t^j is

$$p_t^j z_t^j = q_t^j \tilde{\beta} \mathbf{E}[J_n^j(\mathcal{S}_{t+1}^j)] \tag{13}$$

which states that the opportunity cost in terms of output of transferring one additional worker from production to recruitment (LHS) should be equal to the expected number of workers recruited times their discounted future marginal value to the firm (RHS).

The envelope condition with respect to current period employment n_t^j is

$$J_n^j(\mathcal{S}_t^j) = p_t^j z_t^j (1 - v_t^j) - w_t^j + (v_t^j q_t^j + 1 - \rho^j) \tilde{\beta} \mathbf{E}[J_n^j(\mathcal{S}_{t+1}^j)]. \tag{14}$$

Combining the two equations above yields the equation for the marginal value of labor

$$J_n^j(\mathcal{S}_t^j) = p_t^j z_t^j \left(1 + \frac{1 - \rho^j}{q_t^j}\right) - w_t^j, \tag{15}$$

as well as the inter-temporal equation determining the optimal choice of the share of recruiters

$$p_t^j z_t^j = q_t^j \tilde{\beta} \mathbf{E} \left[p_{t+1}^j z_{t+1}^j \left(1 + \frac{1-\rho^j}{q_{t+1}^j} \right) - w_{t+1}^j \right]. \tag{16}$$

The above equation is similar to the free entry condition in the standard single firm-worker DMP model to the difference that the cost of creating vacancies is measured in units of labor productivity instead of units of output.

4.5 Wage determination

I assume real wages, in units of tradable goods, are determined at the beginning of each period through Nash bargaining between firms and households. The Nash bargaining solution requires that the wage maximizes the generalized Nash product

$$V_{n^j}(\mathcal{S}_t)^{\phi} J_{n^j}^j(\mathcal{S}_t^j)^{(1-\phi)}$$

where ϕ represents the household's (worker's) bargaining power. The resulting first order condition is

$$V_{n^j}(\mathcal{S}_t) = \frac{\phi}{1-\phi} \frac{U_c(c_t, n_t)}{p_t} J_{n^j}^j(\mathcal{S}_t^j).$$

Plugging the current and next period versions of the above expression into (11) and using (13) and (15) yields the wage equation

$$w_t^j = \phi p_t^j z_t^j \left(1 + \theta_t^j \right) - (1-\phi) p_t \frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} \tag{17}$$

which can be interpreted as a weighted average of the MPL of the worker, including both his contribution to production as well as the recruiting effort the firm saved by hiring him, and his MRS.

4.6 Commodity exports

I model the commodity producing sector by assuming a fixed endowment of commodity goods y_t^C supplied to the rest of the world for an exogenously fixed price p_t^C, i.e. the price (terms of trade) of commodity exports relative to tradable goods, which follows the stochastic process

$$\log p_{t+1}^C = (1-\rho_C)\log \bar{p}^C + \rho_C \log p_t^C + \sigma_C \varepsilon_{t+1}^C.$$

Revenues from commodity exports in terms of the numeraire are denoted

$$\Pi^C = p_t^C y_t^C$$

and are directly distributed to households in a lump-sum fashion every period.

4.7 Equilibrium

The resource constraint for the non-tradable goods sector

$$y_t^N = c_t^N \tag{18}$$

has to hold in every period. The trade balance is defined as the difference between domestic output and domestic absorption of tradable and commodity goods

$$tb_t = y_t^T - c_t^T + p_t^C y_t^C.$$

The current account, i.e. the sum of the trade balance and the net investment income, is equal to the change in the net foreign asset position

$$ca_t \equiv r_t a_t + tb_t = a_{t+1} - a_t. \tag{19}$$

The combination of the assumptions of incomplete asset markets, an exogenous discount factor and an exogenous cost of borrowing in international markets result in the model being non-stationary. In order to impose stationarity, I follow Schmitt-Grohé and Uribe (2003) in assuming that the interest rate the local economy faces in international markets r_t is equal to the world interest rate r^* adjusted for a risk premium which is an increasing and convex function of the net foreign asset position

$$r_t = r^* + \psi[\exp(\bar{a} - a_t) - 1] \tag{20}$$

where $\psi > 0$ and $\bar{a}$ are parameters. The economic intuition behind this device is simple: as domestic agents' savings increase above their steady state level the country risk premium decreases which lowers the interest rate they earn, discourages further saving and encourages consumption. This device captures in a reduced-form way the financial frictions that generate a strong debt-level-sensitivity of the interest rate faced by developing countries in international markets (Uribe and Schmitt-Grohé, 2017).

Definition 1 *Given the exogenous path of sectoral productivities* $\{z_t^N, z_t^T\}_{t=0}^{\infty}$ *and commodity terms of trade* $\{p_t^C\}_{t=0}^{\infty}$*, a stochastic equilibrium is defined as the time paths of consumption* $\{c_t, c_t^N, c_t^T\}_{t=0}^{\infty}$*, assets* $\{a_t\}_{t=0}^{\infty}$*, interest rate* $\{r_t\}_{t=0}^{\infty}$*, prices* $\{p_t, p_t^N\}_{t=0}^{\infty}$*, real wages* $\{w_t^N, w_t^T\}_{t=0}^{\infty}$ *and labor market measures* $\{n_t^N, n_t^T, v_t^N, v_t^T, u_t^N, u_t^T, \theta_t^N, \theta_t^T\}_{t=0}^{\infty}$ *that satisfy in every period* t *the following*

- *Optimal allocation of consumption between non-tradables and tradables* (7)*;*
- *Price index equation* (8)*;*
- *Consumption Euler equation* (9)*;*
- *Sectoral labor market tightness* (1)*;*
- *Laws of motion of sectoral employment* (2)*;*
- *Sectoral allocation of job search* (10)*;*
- *Sectoral job creation equations* (16)*;*
- *Sectoral wage equations* (17)*;*
- *Aggregate labor force equation* (3)*;*
- *Resource constraint for non-tradables* (18)*;*
- *Current account equation* (19)*;*
- *Interest rate equation* (20)*.*

5 Steady state equilibrium and comparative statics

Before delving into the dynamics of the model, it seems useful to have a look at the long run effects of different changes in the economic environment on the equilibrium allocation. To do that, I solve for the steady state equilibrium where output, consumption, employment, asset holdings, prices and wages are all constant in both sectors. A constant sectoral employment in either laws of motion (2) or (12) implies

$$n^j = \frac{\pi^j u^j}{\rho^j} \tag{21}$$

and

$$v^j = \frac{\rho^j}{q^j}$$

which are equivalent given the definition of θ^j. However, the second equation makes it easy to see that for $v^j \in [0,1]$ to hold, there is an upper bound on θ^j such that $q^j(\theta^j) \geq \rho^j$.

From the first order condition (10) we have

$$\pi^T V_{n^T} = \pi^N V_{n^N} \tag{22}$$

where

$$V_{n^j} = \frac{U_c(c,n) w^j / p_t + U_n(c,n)}{1 - (1 - \rho^j - \pi^j)\beta}.$$

The Job Creation condition (16) can be written as

$$\frac{1}{\beta} = 1 + q^j \left(1 - \frac{w^j}{p^j z^j}\right) - \rho^j \tag{23}$$

where the real wage is given by

$$w^j = \phi p^j z^j \left(1 + \theta^j\right) - (1 - \phi) p_t \frac{U_n(c,n)}{U_c(c,n)}. \tag{24}$$

Given the values of n^j and u^j the definition of the labor force

$$u^T + u^N + n^T + n^N = 1 \tag{25}$$

has to hold. The consumption of non-tradable goods has to satisfy the resource constraint

$$c^N = y^N = z^N n^N (1 - v^N). \tag{26}$$

From the Euler equation (9) and the stationarity assumption (20) we get

$$\frac{1}{\beta} = 1 + r^* + \psi[\exp(\bar{a} - a) - 1]$$

where by assuming $1 + r^* = \frac{1}{\beta}$ I get the steady state net asset position $a = \bar{a}$. It follows that the current account equation (19) simplifies to

$$c^T = y^T + \bar{p}^C y^C + r^* \bar{a} \tag{27}$$

which plays the role of a market clearing condition for the commodity and non-commodity tradable goods.

Definition 2 *For given levels of sectoral productivities $\{z^N, z^T\}$ and commodity price $\bar{p}^C$, a steady state equilibrium is characterized by consumption levels $\{c, c^N, c^T\}$, prices $\{p, p^N\}$, real wages $\{w^N, w^T\}$ and labor market measures $\{n^N, n^T, v^N, v^T, u^N, u^T, \theta^N, \theta^T\}$ that satisfy the following*

- *Optimal non-tradables and tradables consumption* (7)*;*
- *Price index equation* (8)*;*
- *Sectoral labor market tightness* (1)*;*
- *Sectoral labor market clearing conditions* (21)*;*
- *Sectoral allocation of job search* (22)*;*
- *Sectoral job creation equations* (23)*;*
- *Sectoral wage equations* (24)*;*
- *Aggregate labor force equation* (25)*;*
- *Resource constraint for non-tradables* (26)*;*
- *Current account equation* (27)*.*

Since separation is exogenous, equilibrium in the labor market is mainly governed by the job creation decision of firms in each sector. In contrast, households arbitrage between the two sectors by allocating their job seekers as shown in figure 2. The optimal choice of the representative household is to allocate its unemployed members such that the marginal benefit of searching is equalized across sectors. Notice

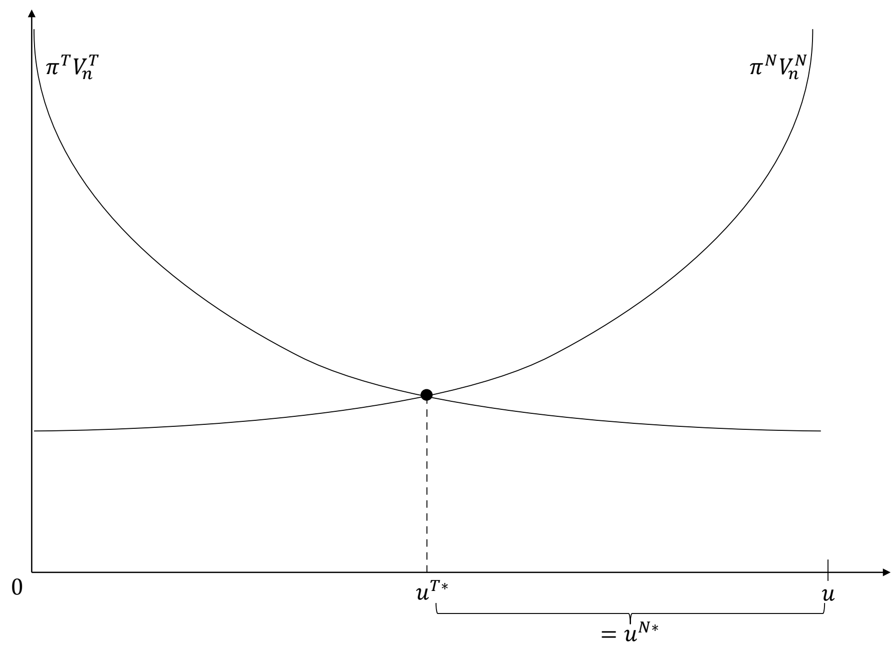


Figure 2: Equilibrium sectoral job search decision.

that $\pi^j V_n^j$ is decreasing in u^j as a result of the congestion externality implied by the decreasing marginal returns of the matching function. An increase in V_n^j will result in the household increasing u^j and decreasing u^{-j} to benefit from the higher utility of having an additional member working in sector j. This will drive π^j down and push π^{-j} up because of the congestion externality between job seekers in each sector.

Figure 3 depicts the equilibrium decision of firms in the (v^j, w^j) space. The behavior of firms in each labor market is governed by the job creation equation (23), labeled as JC^j, and the wage equation (24), labeled as W^j. W^j depicts the wage in sector j as a function of labor market tightness. For a given number of job seekers u^j, the wage equation W^j is increasing in v^j. The intuition is that, as a result of the congestion externality, the higher the number of recruiters competing for the same number of unemployed workers, the higher the search costs. In this case, hiring a worker economizes on these costs. The worker recognizes his additional value to the firm when the labor market is tight and thus bargains for a higher wage. The job creation condition JC^j depicts the number of recruiters v^j as a decreasing function of the wage w^j. A higher wage reduces the profits from hiring an additional worker as seen from equation (14). This in turn implies a lower share of recruiters v^j. For a given level of job seekers in each market u^j, the intersection of JC^j and W^j determines the equilibrium wage and recruitment effort in sector j. Once v^j

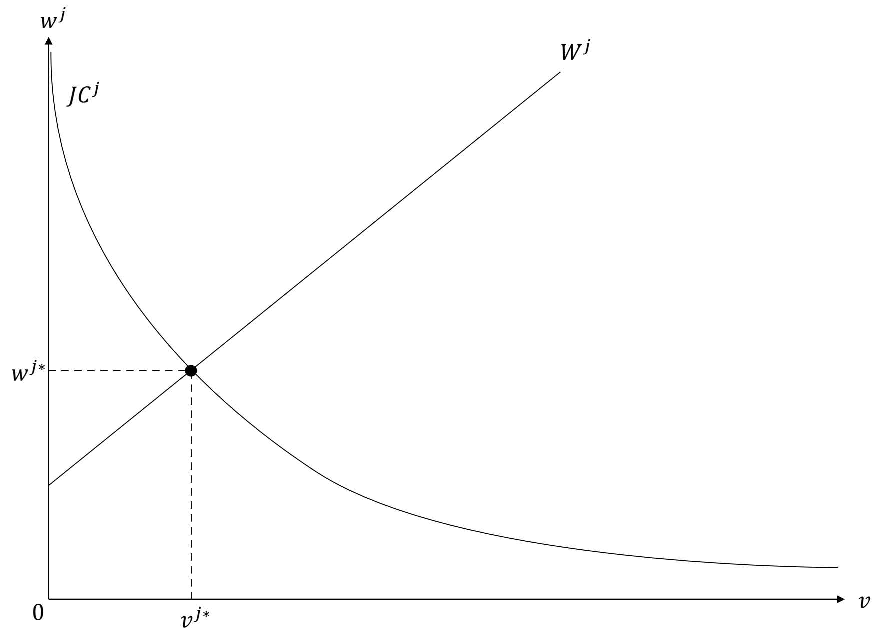


Figure 3: Equilibrium job creation and wage.

and u^j are known, equation (21) determines employment level in sector j as depicted in figure 4.

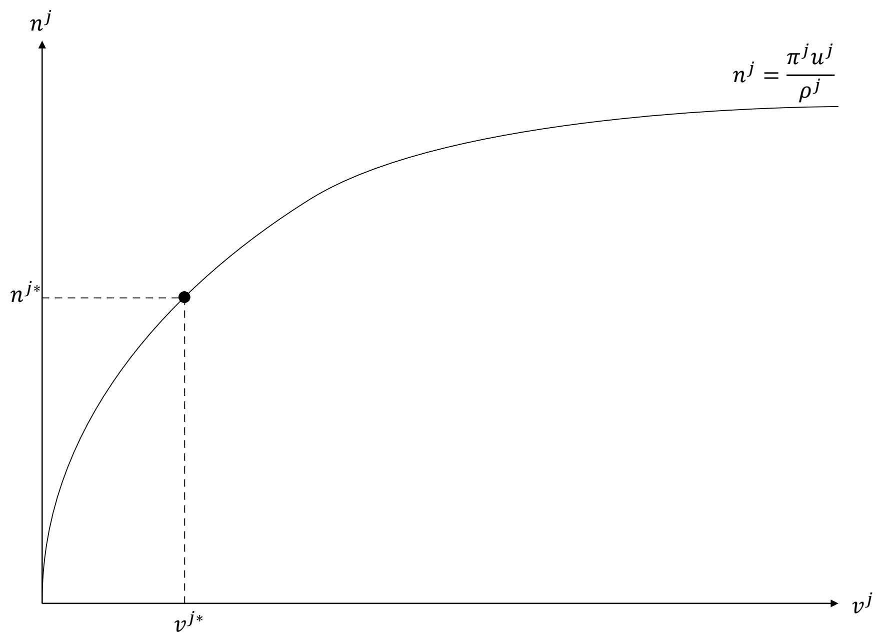


Figure 4: Equilibrium sectoral employment.

Notice that if we set the bargaining power of workers to $\phi = 0$ in the wage equation (24), the wage will equal the MRS as the firm extracts all the surplus of the match. When the MRS is constant, as is the case here, this implies that the wage is constant and hence employment will absorb all changes in the MPL. This can be easily seen in figure 3, by making W^j flat. Shifts in JC^j will have a higher effect on vacancy posting and hence on job creation and employment. If on the contrary $\phi = 1$, W^j is vertical and the wage will react one to one to shocks to the MPL as workers extract all the surplus from the match.

As opposed to the standard one-sector DMP, sectoral labor market tightness does not map directly into unemployment. Since unemployed workers can move seamlessly between the two sectors, there is no sectoral unemployment and no sectoral Beveridge curve per se. θ^j determines sectoral employment which, when summed across sectors, determine the aggregate level of unemployment $u = 1 - n^T - n^N$.

To combine demand and supply of labor in both sectors, first I replace V_n^j and the sectoral wage equations by their expression in equation (22) to get

$$\frac{\pi^T \left(U_c(c,n) z^T (1+\theta^T)/p + U_n(c,n)\right)}{1-(1-\rho^T-\pi^T)\beta} = \frac{\pi^N \left(U_c(c,n) p^N z^N (1+\theta^N)/p + U_n(c,n)\right)}{1-(1-\rho^N-\pi^N)\beta}$$

Next, using (23) to get rid of $p^j z^j$ and replacing with the appropriate functional expressions for the marginal utility of consumption and labor I get

$$\frac{\pi^T b \left(\theta^T + \frac{1/\beta-1-\rho^T}{q^T}\right)}{\left(1-\phi(1+\theta^T) - \frac{1/\beta-1-\rho^T}{q^T}\right)(1-(1-\rho^T-\pi^T)\beta)} = \frac{\pi^N b \left(\theta^N + \frac{1/\beta-1-\rho^N}{q^N}\right)}{\left(1-\phi(1+\theta^N) - \frac{1/\beta-1-\rho^N}{q^N}\right)(1-(1-\rho^N-\pi^N)\beta)}$$

where π^j and q^j are both function of θ^j. The above expression relates the two sectors' labor market tightness to the labor market parameters. It is clear that the relative steady state labor market tightness of the two labor markets is not affected by changes in relative productivities and prices between the two production sectors. This is because households will arbitrage away any differences between the two. This result is similar to the one obtained by Chang (2011) in a closed economy model. However, this need not be true out of the steady state.

In the rest of this section I will focus on two experiments: a permanent increase in the relative productivity of tradable firms $\frac{z^T}{z^N}$ and a permanent increase in the commodity terms of trade $\bar{p}^C$. This will allow me to present some intuition before looking at the stochastic version of the model.

After a permanent increase in the relative productivity of the tradables sector z^T, tradable firms are willing to supply a higher quantity at the same price. As shown in figure 5 this results in non-tradable goods becoming more expensive relative to tradable goods implying an appreciation of the real exchange

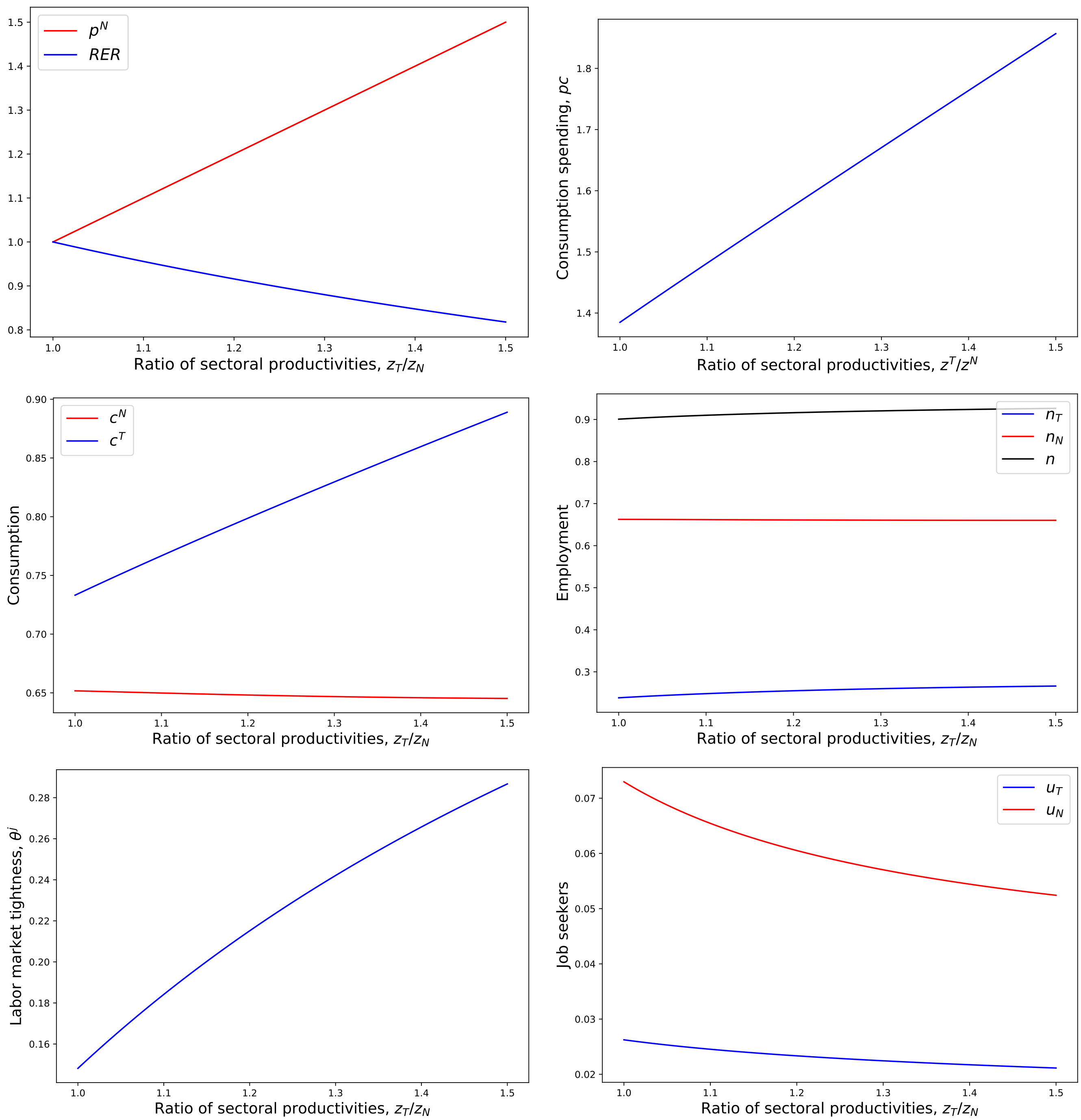


Figure 5: Effect of changing relative sectoral productivity z^T/z^N.

rate. The consumption of tradables increases relative to the consumption of non-tradables. Overall, total consumption spending increases as the economy becomes richer. The increase in productivity leads not only to a reallocation of labor to the production of tradables but an increase in overall employment since aggregate labor productivity is now higher. This is reflected through an increase in labor market tightness in both sectors. Households reallocate their job search from the non-tradables to the tradables sector which is reflected in a smaller fall of u^T compared to u^N as seen in the lower right-hand-side panel of figure 5.

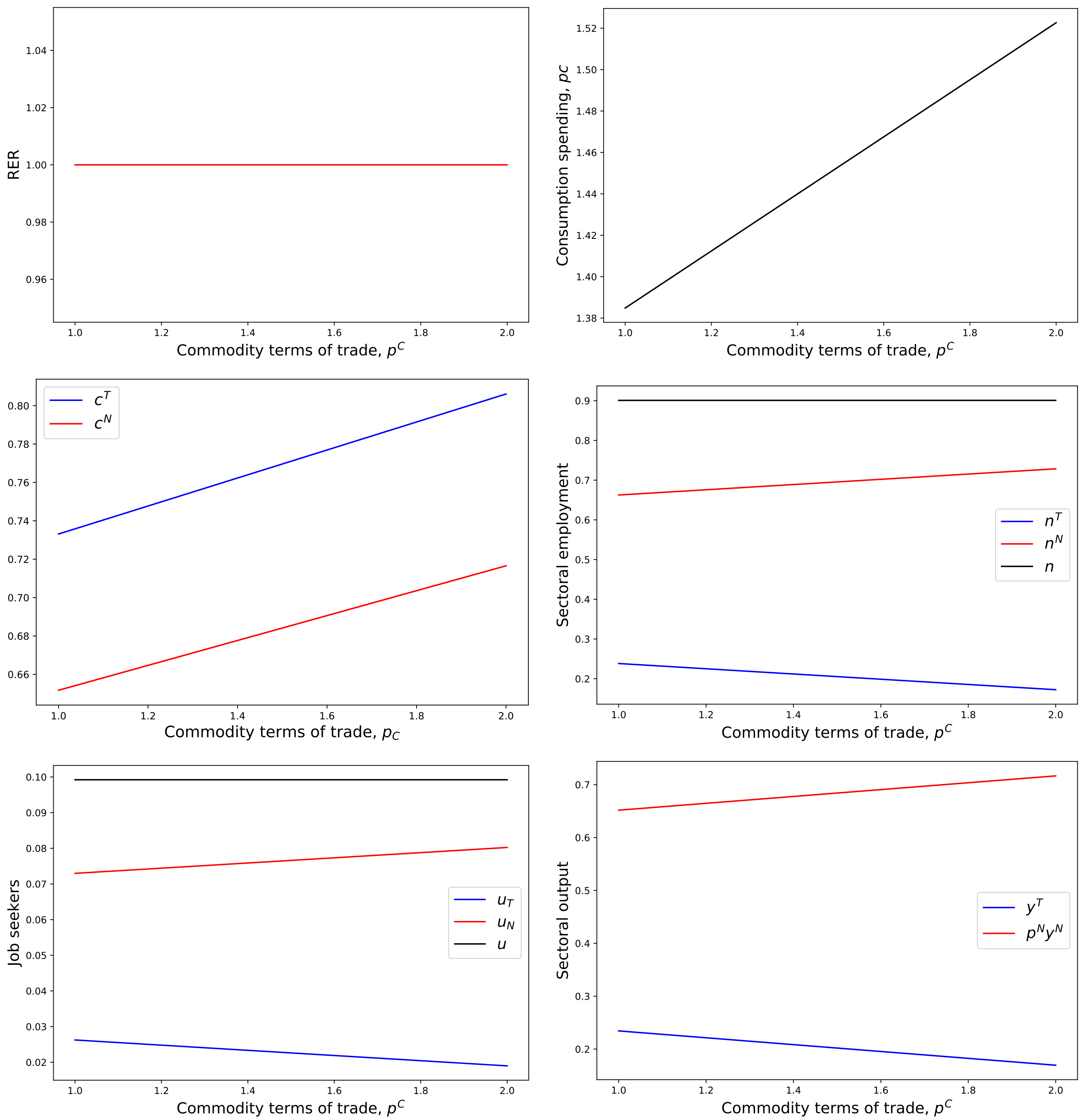


Figure 6: Effect of changing the commodity terms of trade p^C.

Next I analyze the steady state effect of a permanent increase in the price of commodities on sectoral labor reallocation. The effects are shown in figure 6. An increase in $\bar{p}^C$ results in an increase in the wealth of households. Households would like to allocate this increase in wealth between the consumption of tradables and non-tradables in proportion to the ratio of prices. In the steady state the allocation of resources in the economy adjusts such that relative prices stay the same. As a consequence, consumption of both goods will increase in the same proportion. In order to accommodate the increase in the demand for non-tradables, output and employment in the non-tradables sector increase as labor reallocates from the tradables to the non-tradables sector. However, as opposed to the increase in sectoral productivity, total employment and output do not increase as the excess consumption of tradables results in higher imports financed by the commodity windfall. This experiment is an illustration of the Dutch disease as an increase in the commodity exports of the economy induces an appreciation of the real exchange rate which in equilibrium reduces the size of the non-commodity tradable sector.

From the two experiments discussed above we can see that, as opposed to a change in relative sectoral productivity, a permanent increase in the price of commodities does not affect the real exchange rate in the steady state. The labor market always adjusts in the long run in order to absorb the additional demand for non-tradable goods in the economy. In order to assess how these adjustments actually happen and how fast they do we turn in the next section to out-of-steady-state transition dynamics.

6 Calibration and dynamics

In this section I study the out-of-steady-state transition dynamics of the model in reaction to a commodity price shock. In oder to understand the role of labor search frictions in the propagation of the shock throughout the economy, I calibrate and simulate three small open economy models: the baseline business cycle model with labor search frictions (DMP model henceforth), a version of the standard real business cycle model (RBC model), and a real business cycle model with labor search frictions (RBC-DMP model).[3]

6.1 Calibration

The three versions of the model are calibrated for the Chilean economy on a quarterly frequency basis. I define the tradable goods sector y^T to include agriculture and industrial production. In the case of Chile, the commodity sector y^C represents the mining industry. The non-tradable goods sector y^N represents the production of the rest of the economy. The data used covers the period 1999Q4-2016Q4 following the

[3]All three models are solved using a first order approximation around the non-stochastic steady state using Dynare (Adjemian et al., 2017).

Table 2: Parameter values

Param.	Description	DMP	RBC-DMP	RBC	Source/Target
Preferences					
β	Discount factor	0.9902	0.9902	0.9902	4% annual interest rate
σ	Inv. of elast. of inter. subst.	2.00	2.00	2.00	Literature.
φ	Share of NT in consumption	0.4706	0.4168	0.3616	Match $\frac{p^N c^N}{pc} = 0.4706$ (Medina and Naudon, 2011).
ω	T/NT elast. of substitution	0.50	0.50	0.50	Akinci (2011).
η	Wage elasticity of labor supply	-	-	1.455	Uribe and Schmitt-Grohé (2017).
Labor market					
ρ^N, ρ^T	Job separation rate	0.0620	0.0620	-	Marcel and Naudon (2016).
γ	Elast. of matching function	0.4957	0.4957	-	Data (1999Q4-2016Q4).
ϕ	Bargaining power of workers	0.4957	0.4957	-	Hosios efficiency condition.
A	Matching efficiency	1.4506	1.4506	-	Match $\pi = 0.5629$.
b	Unemployment flow value	0.8169	1.4923	-	Match $\frac{b}{MPL/p} = 0.71$ (Hall and Milgrom, 2008).
Production					
α^N	NT capital share parameter	0	0.25	0.25	Uribe and Schmitt-Grohé (2017).
α^T	T. capital share parameter	0	0.35	0.35	Uribe and Schmitt-Grohé (2017).
δ	Capital depreciation	-	0.0260	0.0260	10% annual rate.
y^C	Commodity output	0.1378	0.4240	6.4822	Match $\frac{p^C y^C}{py} = 13.46\%$, Data (1999Q4-2016Q4).
Aggregate shocks					
$\bar{z}$	TFP process steady state level	1	1	1	Normalization.
ρ_z	TFP process persistence	0.8064	0.8064	0.8064	Data (1999Q4-2016Q4).
σ_z	TFP process s.d.	0.0085	0.0085	0.0085	Data (1999Q4-2016Q4).
$\bar{p}^C$	Comm. ToT steady state level	1	1	1	Normalization.
ρ_{p^C}	Comm. ToT process persistence	0.8472	0.8472	0.8472	Data (1999Q4-2016Q4).
σ_{p^C}	Comm. ToT process s.d.	0.0905	0.0905	0.0905	Data (1999Q4-2016Q4).
Asset markets					
ψ	Parameter of the risk premium	0.001	0.001	0.001	Schmitt-Grohé and Uribe (2003).
$\bar{a}$	Steady state net asset position	36.6330	76.5053	1.7801e+03	Match $\frac{n^T}{n} = 73.55\%$.
r^*	World interest rate	0.01	0.01	0.01	4% annual interest rate.

liberalization of the exchange rate regime. Table 2 summarizes the calibration. More details are given in what follows. Most of the parameters are present in both models except those pertaining to the labor search frictions, which are specific to the DMP and RBC-DMP models, and the wage elasticity of labor supply η which is specific to the RBC model.

Preferences: The discount factor β is set at 0.9902 based on a steady state annual interest rate of 4%. I set σ, which determines both risk aversion and the elasticity of intertemporal substitution, to a value of 2 following Shimer (2010). The wage elasticity of labor supply in the RBC model η is set to 1.455 following Uribe and Schmitt-Grohé (2017). The parameter φ is set in each model such that the share of non-tradables in the consumption basket

$$\frac{p^N c^N}{pc} = 47.06\%$$

as provided by Medina and Naudon (2011). Following the literature review by Akinci (2011), the elasticity of substitution between tradables and non-tradable goods in the consumption basket ω is set to 0.5. This means that a 1% increase in the relative price of non-tradables results in a 0.5% fall in the relative consumption of non-tradables as households substitute tradables for non-tradables in their total consumption.

Labor market: The labor market side of the DMP and RBC-DMP models includes the following parameters: $b, \rho^N, \rho^T, \phi, A, \gamma$. Except for the disutility of labor b, the steady state labor market block can be calibrated separately from the rest of the model. At the steady state

$$u = 1 - n^T - n^N = 1 - \frac{\pi^T u^T}{\rho^T} - \frac{\pi^N u^N}{\rho^N}$$

where I replaced n^j using equation (21). In the absence of sector-specific data, I assume that the tradable and non-tradable sectors have the same average rates as the overall economy. Setting $\pi^T = \pi^N = \pi$ and $\rho^T = \rho^N = \rho$ in the equation above returns

$$u = 1 - \frac{\pi}{\rho}(u^T + u^N) = 1 - \frac{\pi}{\rho} u$$

which can be solved for u to get the standard Beveridge curve

$$u = \frac{\rho}{\rho + \pi} \tag{28}$$

which implies that one can match two out of three moments: the unemployment rate, the job finding rate and the job separation rate. Since the focus here is on business cycle fluctuations I choose to match the average job finding and separation rates and leave out the level of unemployment.

Following Mortensen and Nagypal (2007) I estimate the elasticity of the matching function using aggregate labor market data. Their methodology rests on two assumptions: a constant job separation rate and a fast adjustment of the labor market. First, start from (28) and replace with $\pi = \frac{A(vn)^{\gamma}(u)^{(1-\gamma)}}{u}$. After some algebra one obtains

$$\frac{\partial \log vn}{\partial \log u} = -\frac{1}{\gamma}\left(\frac{u}{1-u} + 1 - \gamma\right).$$

I estimate $\frac{\partial \log vn}{\partial \log u}$ from HP filtered Chilean data on job vacancies and unemployment over the period 1999Q4-2016Q4 using a simple OLS regression. I then plug the resulting regression coefficient in the previous equation, along with average unemployment, and solve it for γ. I obtain a point estimate of

$\gamma = 0.4957$, with a 95% confidence interval of $(0.4370, 0.5725)$, very close to the 0.5 value used by most of the labor search literature. I then set the Nash bargaining parameter $\phi = \gamma$ to satisfy the Hosios (1990) efficiency condition. This leaves us with four parameters, b, ρ^N, ρ^T, A.

Naudon and Pérez (2017) provide measures for the job finding and separation rates for the Chilean economy following the methodology proposed by Shimer (2012). Over the period 1962-2015, they report a monthly job finding rate of 24.6% and a monthly job separation rate of 2.4%. However, their study is focused on the metropolitan region of Santiago which all but excludes the mining industry. I turn instead to Marcel and Naudon (2016) who use data from the Chilean labor force survey to estimate the transition probabilities nation-wide. They report an average monthly job finding rate for the period 1996-2016 of 24.11% and an average monthly job separation rate of 2.11%.[4] In order to convert quarterly rates into monthly rates, I follow the methodology of Shimer (2005) by assuming that the probability of moving from state X to Y follows a Poisson process with arrival rate λ^{XY} such that

$$p^{XY} = 1 - \exp(-\lambda^{XY}).$$

Using the monthly job finding and separation probabilities from Marcel and Naudon (2016) I solve for the monthly arrival rates which I then multiply by 3 to get the corresponding quarterly arrival rates. Using these rates I get a quarterly job finding probability of 56.29% and a quarterly job separation probability of 6.20%. The implied unemployment rate from the Beveridge curve relationship is 9.92%, which is close to 9.00%, the average unemployment rate over the period 1999Q4-2016Q4. This leaves me with two parameters b and A which I use to match $\pi = 56.29\%$ and, following Hall and Milgrom (2008), a ratio of the unemployment flow value to the MPL in both sectors of

$$\frac{b}{MPL^j/p} = 0.71.$$

This calibration results in a job filling rate of $q^j = 3.8$ and a share of recruiters $v^j = 0.0163$. In comparison, Hagedorn and Manovskii (2008) and Silva and Toledo (2009) report evidence from the US that recruiting a worker costs approximately 4% of one worker's quarterly wage meaning one recruiter attracts $q^j = 25$ workers over a quarter, a much higher number.

[4] In the absence of official reconciled figures, I ignore the change in the methodology used for the Chilean labor force survey in 2009 and focus on the period 1996-2016 as a whole.

Production: I match the average share of the non-tradables sector in total employment over the period 1999Q4-2016Q4 of 73.55%. The tradables sector employs the remaining. I calibrate the steady state commodity endowment y^C to target the average share of the mining GDP in total real GDP in Chile of 13.46%. I set the annual depreciation rate of capital at 10% for both sectors which translates into a quarterly depreciation rate of 2.6%. Parameters α^N and α^T are used to target labor and capital shares in each sector's production. To see that start from the sectoral production function to get

$$\frac{\partial y^j}{\partial k^j} = y^j_k = \alpha^j \frac{y^j}{k^j}$$

which can be solved for the share of capital in output

$$\frac{y^j_k k^j}{y^j} = \alpha^j.$$

The 2013 Chilean input-output table implies the implausibly high values of $\alpha^T = 0.6337$ and $\alpha^N = 0.5122$. I follow instead Uribe and Schmitt-Grohé (2017) by setting $\alpha^T = 0.35$ and $\alpha^N = 0.25$ in line with most of the literature.

Aggregate shocks: The parameters of the exogenous process for productivity and commodity terms of trade are estimated separately from the Chilean data using OLS. The steady state levels are normalized to 1. The stochastic process for p^C is estimated using mining terms of trade, i.e. the ratio of the mining exports price index to the imports price index.[5]

Asset markets: I set the steady state interest rate faced by the local economy in international markets at $r^* = 1.00\%$ which corresponds to an annual interest rate of 4%. The parameter ψ is set at a value of 0.001, the minimum that guarantees a stationary solution. $\bar{a}$ is used to match the share of the non-tradable sector in total employment.

6.2 Dynamics following a commodity terms of trade shock

In general, an increase in the relative price of exports induces both a substitution effect and an income effect. Households substitute away from the consumption of exportable goods in favor of the consumption

[5] In the model p^C corresponds to the relative price of the commodity good in terms of the tradable good.

of imported and non-traded goods. At the same time, the increase in export prices results in a positive income effect (increase in income) that increases the demand for both tradable and non-tradable goods. Both the substitution and income effects result in an increase in the price of non-tradable goods as domestic producers increase their prices. Since in my setting commodities don't enter the consumption basket of households, the substitution effect is absent and only the income effect is active. This means in theory that the impact of a terms of trade shock will be lower.

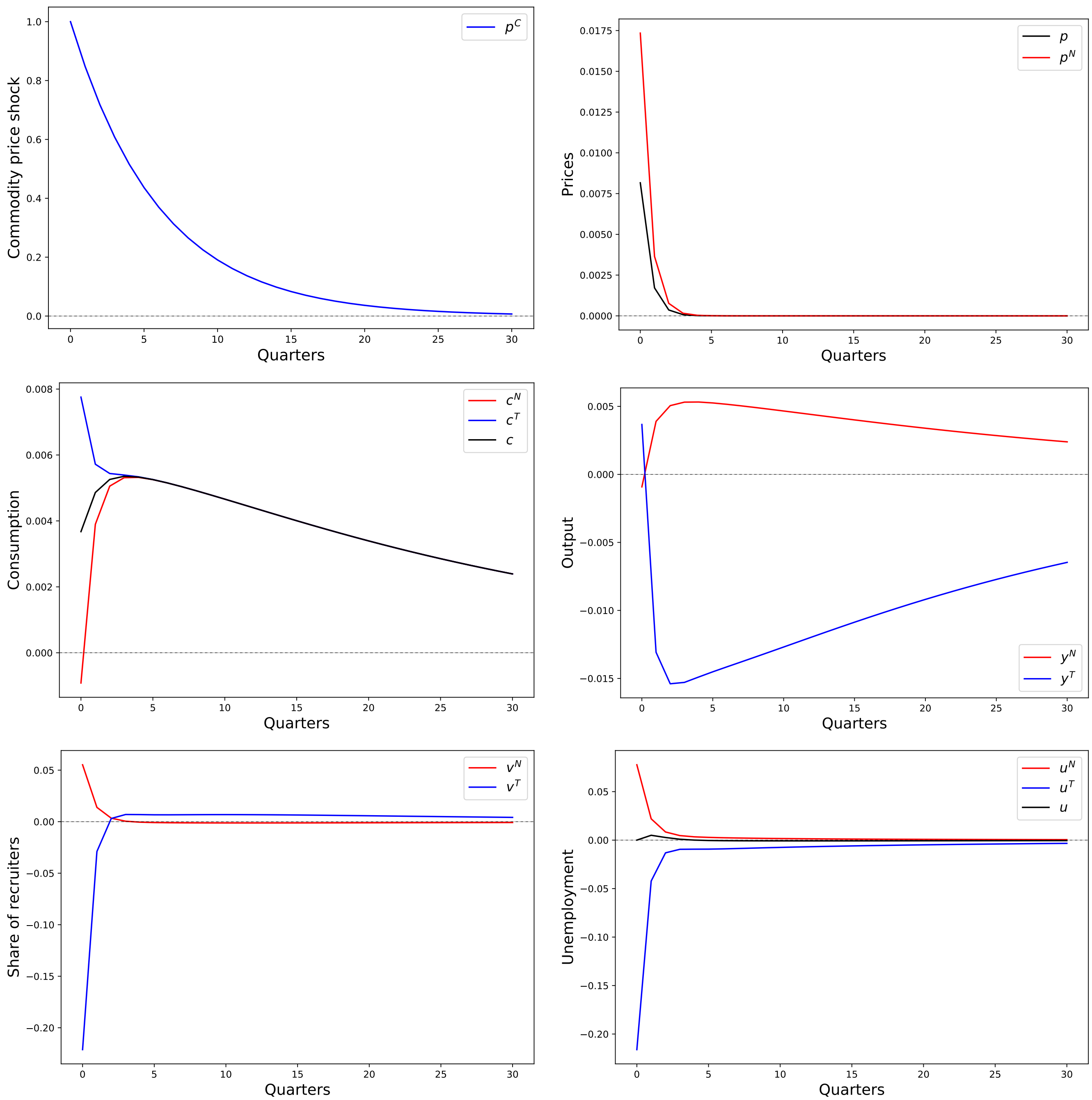
In what follows I discuss more in details the transition dynamics following a commodity terms of trade shock. First I look at the impulse response functions generated by the baseline DMP model. Second, I look deeper at the role of labor search frictions by comparing the impulse response functions generated by three different models.

6.2.1 Impulse response functions of the baseline model

The plots in figures 7 and 8 depict the impulse response functions to a positive commodity terms of trade shock.[6] As households become temporarily wealthier, they demand more of both tradable and non-tradable goods. A higher demand for tradables can readily be satisfied through higher imports while the demand for non-tradables is constrained by the local supply. This excess demand of non-tradable goods exercises an upward pressure on p^N as it takes time for supply to adjust. As a consequence, both p^N and p jump on impact and remain above their steady state value for a couple of quarters. A higher price of non-tradable goods relative to tradable goods results in a temporary appreciation of the real exchange rate as the economy becomes more expensive compared to the rest of the world. As long as p^N remains above its steady state value, households partially substitute away from non-tradable consumption to tradable consumption. Non-tradable firms face now a higher price which prompts them to increase their demand for labor to produce more goods. However, because of the search and matching frictions it takes time for the adjustment to happen. In addition, non-tradable firms face an interesting trade-off: on the one hand it's a great time to produce which means shifting workers from recruitment to production, on the other hand the increase in p^N is persistent and it will be good to shift workers from production to recruitment to increase the future workforce. The persistence in the effect of the shock from the demand side comes from the process itself but also through the consumption smoothing of households via saving in international assets as seen in figure (8). This trade-off between production and recruitment is similar

[6]The shock is an unexpected and temporary 1% increase in the price of the commodity endowment.

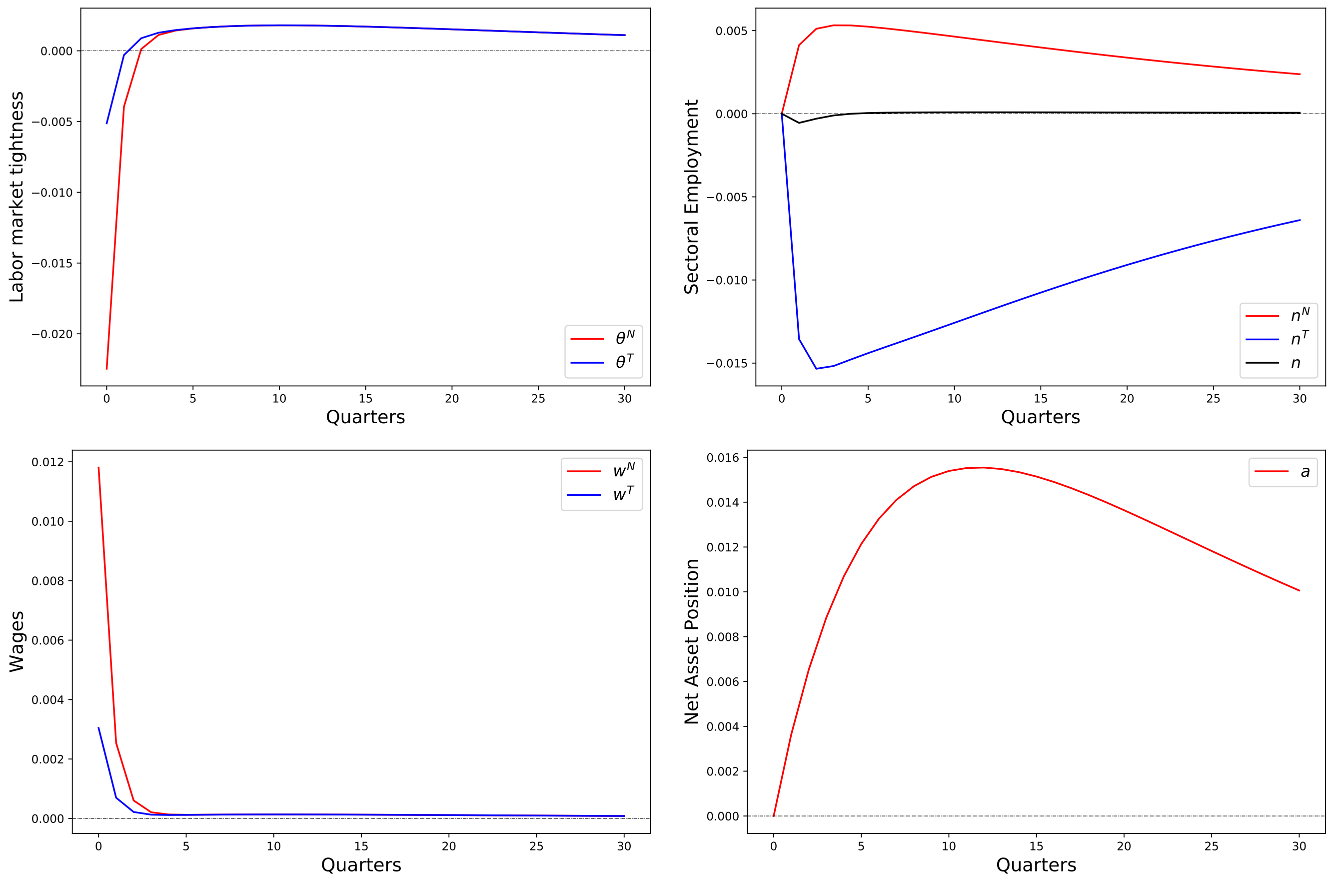
Figure 7: IRFs to a 1% commodity price shock - DMP model (1)



to the one facing firms when deciding to direct a share of output to invest it in capital.

As p^N increases, the surplus of job matches in the non-tradable sector increases which in turn increases w_t^N relative to w_t^T. Households react by reallocating their search effort to the non-tradables labor market as u_t^N increases and u_t^T falls. Since the reallocation of job seekers is faster than the reallocation of recruiters, labor market tightness falls in the non-tradable sector during the first two quarters after

Figure 8: IRFs to a 1% commodity price shock - DMP model (2)



the shock but recovers once the number of recruiters $v^N n^N$ catches up.[7] As the economy becomes more expensive, i.e. higher p, workers demand an increase in their wage to accept forgoing a higher unemployment insurance pb. This can be seen through the wage equation (17) where p_t multiplies the MRS. This effect alone explains the increase in w_t^T which can be seen in lower left-corner panel in figure 8. However, the increase in w_t^T is much less pronounced compared to the increase in w_t^N.

Despite a slight increase due to search frictions in the labor market, total unemployment remains unchanged as a reaction to the additional wealth. This is because of the strong negative cross-sectoral correlation of employment.

As discussed in Corden and Neary (1982), the reallocation of resources between the tradable and non-tradable sectors following a terms of trade shock will depend both on the elasticity of substitution between the domestic tradable and non-tradable goods and the elasticity of substitution between the domestic tradable and foreign tradable (imported) goods. Here, domestic and foreign tradable goods

[7] u^j is a jump variable while $v^j n^j$ depends in part on n^j which is persistent.

are by construction perfect substitutes. The elasticity of substitution between tradable and non-tradable goods is set at $\omega = 0.5$ and as such a 1% increase in p_t^N results only in 0.5% fall in the consumption of non-tradables. This real exchange rate effect is however short-lived as the economy adjusts its sectoral allocation within the first four quarters. What remains is a pure wealth effect as households continue to enjoy the persistent commodity windfall through a higher aggregate consumption.

Regarding the persistence of the effects, the overall economy has two ways of smoothing the temporary increase in wealth: investing in international asset holdings, and changing the allocation of workers from production to recruitment activities. In a closed economy, only the latter mechanism is active. We would expect then that the lower the degree of international risk sharing the higher the volatility of recruitment activities as a share of total employment in response to external shocks.

6.2.2 Model comparison

To disentangle the contribution of labor search frictions to the aggregate and sectoral transmission of terms of trade shocks, I compare three models: the baseline DMP model, the RBC-DMP model which extends the DMP model by adding capital, and the standard RBC model without labor search frictions.[8]

Figures 9 and 10 present a comparison of different impulse response functions from the three models to a 1% unexpected and temporary increase in the commodity price p^C. The reaction of total output py is very similar for all three models as most of it is driven by the increase in commodity output following the shock.

The appreciation of the real exchange rate is more amplified and much more persistent in the RBC-DMP model compared to the DMP and RBC models as seen in figure 9. The RBC-DMP model exhibits the most smoothness and persistence in aggregate consumption after the shock. This is because among the three model economies, the RBC-DMP model has the highest number of ways to transfer wealth across time: capital, international asset holdings and the allocation of workers from production to recruitment.

Overall, the presence of capital in the RBC and RBC-DMP models makes both the impact and the propagation of the shock stronger, in line with the findings of the real business cycle literature in the case of productivity shocks. In the RBC-DMP model, capital accumulation is very similar to recruitment as it allows the inter-temporal transfer of productive resources.[9]

[8] The RBC and RBC-DMP models are described in more detail in appendices A and B.

[9] The only difference between the two is that shocks to the production technology affect also the production of capital while the efficiency of the recruitment technology remains unaffected.

Figure 9: Aggregate IRFs to a 1% commodity price shock - Model Comparison

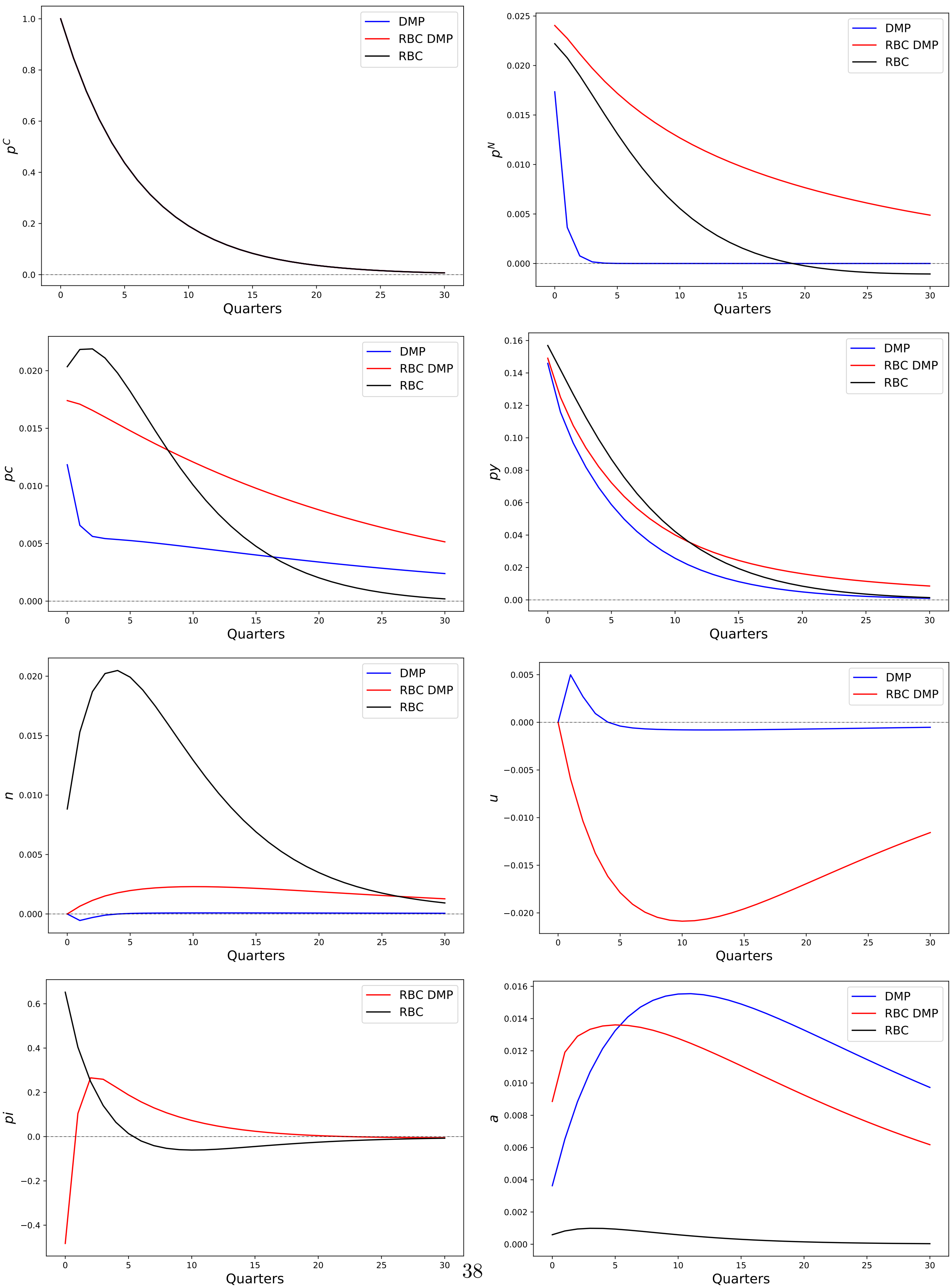
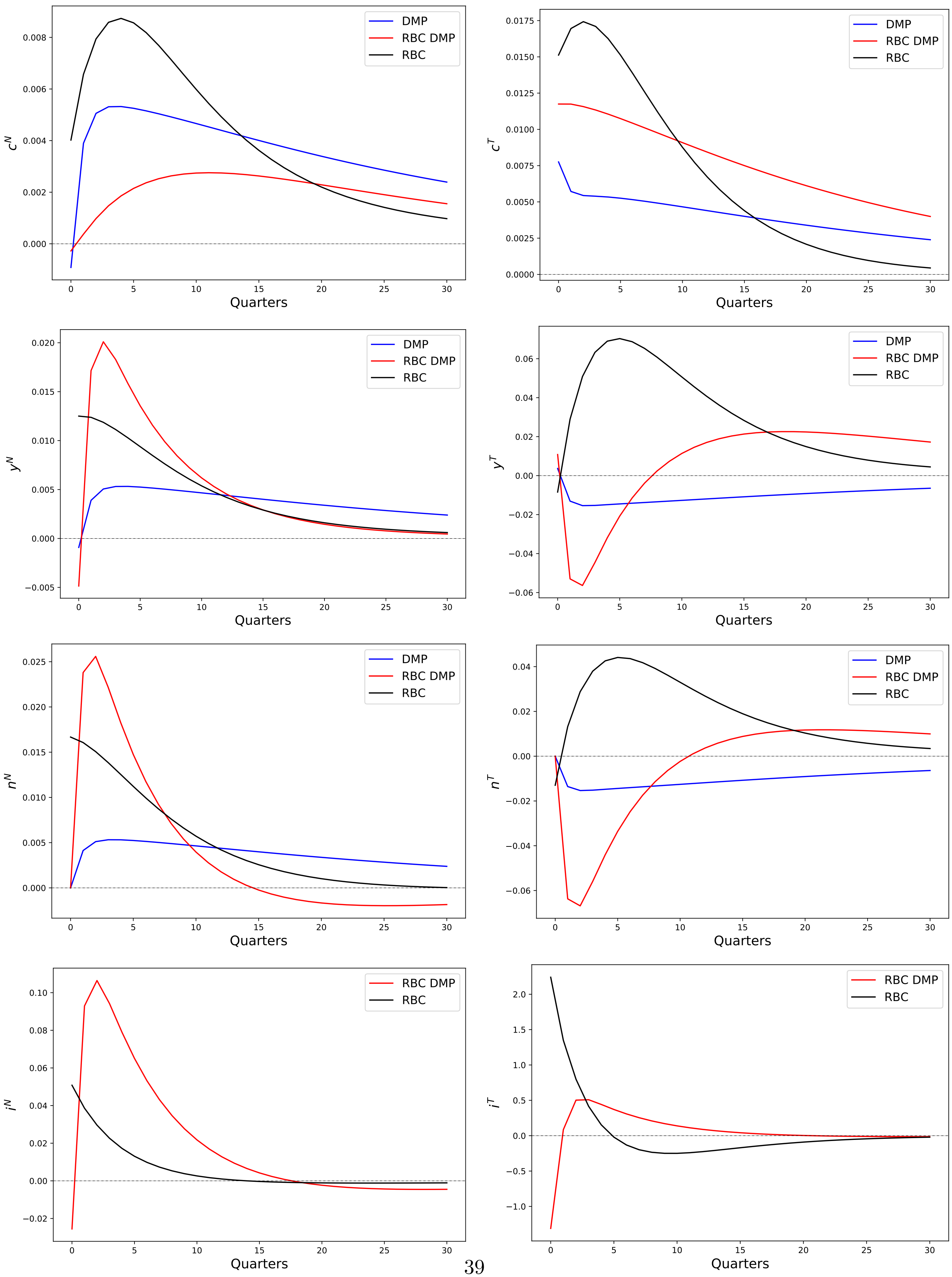


Figure 10: Sectoral IRFs to a 1% commodity price shock - Model Comparison (1)



In the RBC model, almost all sectoral variables jump positively in reaction to the shock. Cross-sectoral correlation is positive for consumption, output, employment and investment. This is the case as the two types of goods are imperfect substitutes and there are no sectoral trade-off occurring in terms of resource allocation. In contrast, the models with labor search frictions exhibit a negative cross-sectoral correlations for output, employment and to lesser extent investment, as labor reallocates from the tradable to the non-tradable sector.

As a result of the presence of search frictions in the labor market, the reaction of employment is much more subdued in the DMP and RBC-DMP models compared to the frictionless RBC model. In the latter, aggregate employment jumps on impact and exhibits a hump-shaped form. The increase on impact occurs in the non-tradables sector. The tradable sector takes more time to catch up but the percentage increase is higher. Compared to the DMP model, the RBC-DMP model exhibits a much stronger reaction of unemployment to the shock. Unemployment falls gradually until it reaches its minimum level between the fifth and tenth quarter, then it returns slowly to its steady state level. This implies a negative correlation between unemployment and commodity terms of trade in line with the data. The presence of capital is hence essential for this feature.

International asset holdings increase as households attempt to save part of the windfall in order to smooth their consumption. However, this increase is much more subdued in the case of the RBC model as domestic absorption is higher due to the spike in investment.

6.3 Business cycle moments: models vs. data

In what follows I compare the quantitative implications of the three models in terms of business cycle moments with their empirical counterpart. In order to make theoretical and empirical moments comparable, both sets of variables are expressed in log and detrended as standard in the literature.[10] I start first with the overall performance of the models subject to both productivity and commodity terms of trade shocks. Afterwards, I focus on the contribution of commodity terms of trade shocks to output, consumption, employment and investment. Finally, I look at how these shocks affect unemployment volatility under different calibrations.

Table 3: Business cycle moments: both shocks

	Data	DMP	RBC-DMP	RBC
Standard deviation				
Mining ToT	0.1713	0.1127	0.1127	0.1127
Productivity	0.0143	0.0103	0.0103	0.0103
Real GDP	0.0167	0.0173	0.0186	0.0244
Private Consumption	0.0218	0.0029	0.0016	0.0081
Investment	0.0603	-	0.2999	0.1519
Unemployment	0.0810	0.0179	0.0151	-
Employment in Non-Tradables	0.0136	0.0055	0.0059	0.0127
Employment in Tradables	0.0226	0.0226	0.0217	0.0202
Rel. standard deviation				
Mining ToT	10.265	6.5145	6.0591	4.6189
Productivity	0.8578	0.5954	0.5538	0.4221
Private Consumption	1.3081	0.1676	0.0860	0.3320
Investment	3.6118	-	16.124	6.2254
Unemployment	4.8534	1.0347	0.8118	-
Employment in Non-Tradables	0.8125	0.3179	0.3172	0.5205
Employment in Tradables	1.3573	1.3064	1.1667	0.8279
Corr. with output				
Mining ToT	0.6470	0.8849	0.8106	0.7082
Productivity	0.6851	0.3911	0.5593	0.6888
Private Consumption	0.9288	0.4353	0.7889	0.8205
Investment	0.7963	-	0.1787	0.8282
Unemployment	-0.6986	-0.4264	-0.4481	-
Employment in Non-Tradables	0.5892	-0.3435	-0.0380	0.8000
Employment in Tradables	0.4161	0.3561	0.2079	0.6623
Corr. with mining terms of trade				
Real GDP	0.6470	0.9271	0.8753	0.7844
Private Consumption	0.7579	0.1652	0.4340	0.1798
Investment	0.3814	-	-0.0281	0.3866
Unemployment	-0.5608	0.0172	-0.0043	-
Employment in Non-Tradables	0.4764	0.0949	0.5458	0.1635
Employment in Tradables	0.3473	-0.0715	-0.4000	0.0366

6.3.1 Overall performance

Table 3 compares some business cycle moments from the data with those generated by the three models. The simulations use both the productivity and the mining terms of trade shocks.

Comparing standard deviations between the data and the three models, one can notice the usual shortcomings reported in the business cycle literatures. Consumption is one order of magnitude less volatile in the models reflecting too much consumption smoothing relative to the data. This is in contrast to Boz et al. (2015) where the labor search model is able to generate a consumption volatility higher than

[10] I use the standard (two-sided) HP filter with parameter 1600.

that of output by adding interest rate shocks. Actually, consumption in the RBC-DMP model is the least volatile as the overall economy has the highest number of means to shift consumption across time: physical capital, international assets and the reallocation of workers from production to recruitment. In contrast with consumption, the volatility of investment is much higher than in the data. This is a standard result in business cycle models and has motivated the literature to introduce convex capital adjustment costs in the RBC model to better match the data. Interestingly enough, investment in the RBC-DMP model is more volatile than in the RBC model suggesting some interaction between labor search frictions and capital. The volatility of unemployment generated by the labor search models falls short of its empirical counterpart which is similar to Shimer (2005) findings for the US. Both in the data and the model the volatility of mining terms of trade relative to output is very high. However, all three models fall short of replicating the much higher empirical counterpart.

Qualitatively, all three models do a good job in replicating the signs of the correlations with output observed in the data. In particular, unemployment is negatively correlated with output in the DMP and RBC-DMP models. The only exception is the correlation of non-tradables employment and output which returns the wrong sign in the labor search models. The RBC model does a better job in this regard. Correlations of output and consumption with the mining terms of trade exhibit the correct sign. Again, the RBC model does a better job regarding the correlation of investment and sectoral employment with the mining terms of trade. Both the DMP and RBC-DMP models generate a very weak correlation with unemployment compared to the data. This is the case as mining terms of trade shocks contribute very little to the variance of aggregate unemployment as we will see later. The labor search models deliver the correct sign of the correlation with employment in the non-tradables sector but fail with the tradables employment. This is a result of adjustments over the inter-sectoral margin in reaction to changes in the real exchange rate by shifting employment between sectors. As we've seen with the IRFs, this results in the negative cross-sectoral correlation of employment and output that tends to cancel out at the aggregate level resulting in behavior at odd with the data.

There are at least three potential solutions to this issue: adding an intensive margin in terms of hours worked by individual as in the RBC model, adding a participation margin such that the labor force shifts in reaction to shocks and finally including an inter-sectoral reallocation friction that imposes a convex adjustment cost to households when shifting job seekers from one sector to another.

Table 4: Business cycle moments: Mining ToT shocks only

	Data	DMP	RBC-DMP	RBC
Standard deviation				
Mining ToT	0.1713	0.1127	0.1127	0.1127
Real GDP	0.0167	0.0153	0.0151	0.0174
Private Consumption	0.0218	0.0006	0.0008	0.0017
Investment	0.0603	-	0.0590	0.0667
Unemployment	0.0810	0.0005	0.0019	-
Employment in Non-Tradables	0.0136	0.0006	0.0033	0.0021
Employment in Tradables	0.0226	0.0018	0.0088	0.0058
Share of empirical SD				
Mining ToT	1	0.6579	0.6579	0.6579
Real GDP	1	0.9162	0.9042	1.0419
Private Consumption	1	0.0275	0.0367	0.0780
Investment	1	-	0.9784	1.1061
Unemployment	1	0.0062	0.0235	-
Employment in Non-Tradables	1	0.0441	0.2426	0.1544
Employment in Tradables	1	0.0796	0.3894	0.2566

Table 5: Variance decomposition (in percentage)

	DMP		**RBC-DMP**		**RBC**	
	z	p^C	z	p^C	z	p^C
Real GDP	21.68	78.32	34.15	65.85	49.22	50.78
Private Consumption	95.69	4.31	73.85	26.15	95.56	4.44
Investment	-	-	96.13	3.87	80.71	19.29
Employment/Unemployment	99.92	0.08	98.50	1.50	96.72	3.28
Employment in Non-Tradables	98.73	1.27	69.32	30.68	97.19	2.81
Employment in Tradables	99.36	0.64	83.41	16.59	91.83	8.17

6.3.2 Mining terms of trade shocks

In this experiment, I turn-off productivity shocks and keep only mining terms of trade shocks in order to see how much the latter contributes to the volatility of the economy. Table 4 compares the standard deviation of some observables with the ones generated by the model. The mining terms of trade shocks contribute significantly to the volatility of output in all three models. This is because the latter includes the mining output and hence is directly affected by the shocks. However, the ratio of theoretical to empirical volatilities of output is lower under the DMP and RBC-DMP models. This confirms that labor search frictions act as a mechanism that dampens the effect of the mining terms of trade shocks by slowing the adjustment of labor compared to the frictionless economy.

The variance contribution of mining terms of trade shocks is lower in the frictionless RBC model compared with the two models with labor search frictions. Fluctuations in commodity prices contribute

Table 6: Comparing different calibrations of the RBC-DMP model - Mining ToT shock only

Standard deviation	**1999Q4-2016Q4**	$\frac{p_t b}{MPL^j} = 0.40$	$\frac{p_t b}{MPL^j} = 0.71$	$\frac{p_t b}{MPL^j} = 0.96$
Real GDP	0.0167	0.0153	0.0151	0.0150
Unemployment	0.0810	0.0006	0.0019	0.0127
Employment in Non-Tradables	0.0136	0.0026	0.0033	0.0044
Employment in Tradables	0.0226	0.0070	0.0088	0.0110

substantially to fluctuation in sectoral employment in particular in the non-tradables sector under the RBC-DMP model. However, there is very little volatility in aggregate employment. Productivity shocks explain almost all the variance in aggregate consumption in both the DMP and RBC models. This is different in the RBC-DMP model where mining terms of trade shock explain around a third of consumption volatility. Overall, the RBC-DMP model gives a higher contribution to mining terms of trade shocks. This confirms that fluctuations in the real exchange rate resulting from terms of trade shocks matter for the sectoral allocation of labor under labor search frictions especially and that this result is particularly amplified when capital is included in the model.

6.3.3 Unemployment volatility

The seminal work of Shimer (2005) has started a decade-long debate on the transmission of productivity shocks to unemployment in models with labor search frictions (Hall, 2005; Mortensen and Nagypal, 2007; Shimer, 2010; Hall and Milgrom, 2008; Hagedorn and Manovskii, 2008; Rogerson and Shimer, 2011). This debate was arguably closed by Ljungqvist and Sargent (2017) who provide an elegant explanation based on the fundamental surplus fraction, i.e. an upper bound on the share of a job's output that the market can allocate to vacancy creation. The authors show that a smaller fundamental surplus fraction results in a higher elasticity of market tightness to productivity shocks.

In the DMP and RBC-DMP models, the fundamental surplus fraction in each sector is

$$\frac{MPL_t^j - p_t b}{MPL_t^j}$$

where MPL^j is the marginal productivity of labor, including economies on search costs, and $p_t b$ is the flow value of unemployment in units of tradable goods. Table 6 presents results from different calibrations targeting three levels of the ratio

$$\frac{p_t b}{MPL_t^j}$$

discussed in the literature: a value of 0.40 proposed by Shimer (2005), 0.71 proposed by Hall and Milgrom (2008) and finally 0.96 proposed by Hagedorn and Manovskii (2008). In each calibration, I adjust the efficiency parameter A in order to match the job finding rate observed in the data. As shown in table 6, the volatility of unemployment is increasing in the steady state ratio of the flow value of unemployment to the MPL in line with the findings of Ljungqvist and Sargent (2017). In particular, going from a value of 0.40 to 0.96 increases the volatility of unemployment by two orders of magnitude. The increase in the volatility of sectoral employment is less spectacular in comparison. Interestingly enough, the volatility of output is slightly decreasing.

A positive shock to commodity terms of trade increases the real exchange rate which increases the fundamental surplus in the non-tradable sector and leads to higher job creation. For a given size of the shock, the smaller the fundamental surplus fraction the larger the elasticity of matches and unemployment to the shock as shown by Ljungqvist and Sargent (2017). A novel effect here is that as the price of the composite consumption basket increases following the shock the relative value of unemployment insurance $p_t b$ increases which makes the fundamental surplus fraction even smaller. The latter effect should amplify the effect of terms of trade shocks on unemployment compared to aggregate productivity shocks in a closed economy.

7 Conclusion

In this paper I have analyzed the impact of terms of trade shocks on a small open economy in the presence of search and matching frictions in the labor market. I used shocks to commodity prices as a proxy in order to remedy the potential endogeneity of changes in general terms of trade highlighted by Schmitt-Grohé and Uribe (2018). The main methodological contribution of the paper is to extend the labor search model with the large family structure (Merz, 1995; Andolfatto, 1996; Shimer, 2010) to an open economy setting with tradable and non-tradable sectors. The presence of two labor markets with search and matching frictions allow me to study the impact of shocks on the intra and inter-sectoral dynamics of labor as in Chang (2011).

As opposed to standard terms of trade shocks, commodity terms of trade shocks operate through the income effect and result in a temporary appreciation of the real exchange rate. The higher price of non-tradable goods relative to tradable goods shifts labor and production from the tradable sector to the non-tradable sector. I analyze the contribution of search and matching frictions to this mechanism by comparing the performance of three small open economy models: a labor search model as in Merz (1995), Andolfatto (1996) and Shimer (2010), a standard real business cycle model and a real business cycle model with labor search frictions. I calibrate the three models to the Chilean economy and show numerically that labor search frictions improve the ability of the model to match some empirical business cycle moments. Compared to the frictionless model, labor search frictions operate as a dampening mechanism that reduces the volatility of output and increases substantially the persistence of the real exchange rate appreciation. The numerical results show that adding capital to the open economy labor search model is essential for matching the data. In addition to capital and financial assets, vacancy creation offers a third margin to smooth the impact of shocks on the economy which results in a less volatile consumption compared to the standard RBC model.

In line with the findings of Ljungqvist and Sargent (2017) in the case of productivity shocks, I provide numerical evidence that the fundamental surplus fraction, i.e. the share of resources allocated by the market to recruitment activities, plays an important role in the transmission of real exchange rate fluctuations to unemployment. A given terms of trade shock translates into a larger change in unemployment the smaller is the fundamental surplus fraction. However, under a reasonable calibration, the volatility of unemployment is too low compared to the data. This is in part due to the high intra-sectoral volatility which tends to cancel-out at the aggregate level. Such behavior of the model can be explained by the absence of frictions at the inter-sectoral margin. A potential solution is to incorporate labor reallocation costs as in Chang (2011).

The theoretical framework presented in this paper provides a good foundation for tackling issues related to business cycle fluctuations in emerging and developing countries. A potential direction of research is to study the unemployment cost of external shocks under different exchange rate regimes. Schmitt-Grohé and Uribe (2016) study this issue in a model of involuntary unemployment. It would be interesting to revisit their results in a labor search model, which arguably provides stronger micro-foundations.

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Appendix A   Model without search frictions

As opposed to the model with labor search frictions, the labor market in this model is a Walrasian market where firms and households take wages as given. In order to make the results comparable with the open economy literature, I assume labor is divisible in this model. The idiosyncratic unemployment risk that gives rise to employed and unemployed agents is not active and as a consequence there is no need for a large household structure to insure against it. All members are employed and supply n^j hours of labor for the wage w^j prevailing in each sector.

I use the standard GHH preferences with divisible labor

$$U(c_t, n_t^N, n_t^T) = \frac{(c_t - G(n_t^N, n_t^T))^{1-\sigma} - 1}{1-\sigma}$$

with

$$G(n_t^N, n_t^T) = \frac{(n_t^N)^\eta + (n_t^T)^\eta}{\eta}$$

assumed to be separable in sectoral employment.

Under this specification, $\eta > 0$ captures the wage elasticity of labor supply and $\sigma > 0$ the intertemporal elasticity of substitution, while the income elasticity of labor supply is zero as in the frictional model.

Households consumption allocation between tradable and non-tradable goods remains the same as before. The household starts the period with the following state variables $\mathcal{S}_t = \{a_t\}$. The household maximizes the total utility of its members by choosing how much to spend in consumption and save in the internationally traded asset a_t and the share of its members to employ in each sector. This problem can be stated in the following recursive formulation:

$$V(\mathcal{S}_t) = \max_{c_t, n_t^N, n_t^T, a_{t+1}} \{U(c_t, n_t^N, n_t^T) + \beta \mathbf{E}[V(\mathcal{S}_{t+1})]\}$$

subject to the budget constraint

$$p_t c_t + a_{t+1} = w_t^N n_t^N + w_t^T n_t^T + (1 + r_t) a_t + \Pi_t + \Pi_t^C$$

where Π_t and Π_t^C are profits transfered from non-commodity firms and the revenues generated from commodity exports respectively, and w_t^j is the real wage earned by household members working in sector $j \in \{N, T\}$. Notice that in this frictionless economy there is no unemployment.

The consumption-saving decision yields the usual Euler equation

$$\frac{U_c(c_t, n_t^N, n_t^T)}{p_t} = \beta \mathbf{E}\left[(1 + r_{t+1})\frac{U_c(c_{t+1}, n_{t+1}^N, n_{t+1}^T)}{p_{t+1}}\right]. \tag{A.1}$$

From the first order condition for sectoral labor we get the labor supply schedule

$$\frac{w_t^j}{p_t} = -\frac{U_{n^j}}{U_c} = n_t^{j\,\eta-1} \tag{A.2}$$

with wage elasticity $\frac{1}{\eta-1}$. The equation above states that the household will supply hours of labor in each sector until the marginal disutility of labor is equal to the sectoral wage evaluated in terms of marginal consumption. As opposed to the model with labor search frictions, wages are equal to the MRS between leisure and consumption.

As opposed to the model with labor search frictions, all the work force is assigned to production. The representative firm in sector j starts the period with the following state variables $\mathcal{S}_t^j = \{z_t^j, k_t^j\}$ and chooses labor n_t^j and investment i_t^j in order to maximize the present value of its profits

$$J^j(\mathcal{S}_t^j) = \max_{n_t^j, i_t^j} p_t^j y_t^j - w_t^j n_t^j - p_t^j i_t^j + \tilde{\beta}\mathbf{E}[J^j(\mathcal{S}_{t+1}^j)]$$

subject to:

$$\begin{aligned} y_t^j &= z_t^j (k_t^j)^\alpha (n_t^j)^{1-\alpha}; \\ k_{t+1}^j &= (1-\delta)k_t^j + i_t^j; \\ \log z_{t+1} &= (1-\rho_z)\log \bar{z} + \rho_z \log z_t + \sigma_z \varepsilon_{t+1}. \end{aligned}$$

The first order condition with respect to labor is

$$w_t^j = p_t^j (1-\alpha) z_t^j \left(\frac{k_t^j}{n_t^j}\right)^\alpha \tag{A.3}$$

which states that the firm hires workers until the MPL is equal to the wage. Labor market clearing in each sector implies that the labor demanded by firms is equal to the labor supplied by households at the prevailing wage w_t^j.

The optimal level of capital is given by

$$p_t^j = \tilde{\beta}\mathbf{E}\left[p_{t+1}^j \alpha^j z_{t+1}^j \left(\frac{k_{t+1}^j}{n_{t+1}^j}\right)^{\alpha^j - 1} + p_{t+1}^j(1-\delta)\right] \tag{A.4}$$

where capital in sector j is produced using the same production technology as good j.

Definition 3 *Given the exogenous path of sectoral productivities* $\{z_t^N, z_t^T\}_{t=0}^{\infty}$ *and commodity price* $\{p_t^C\}_{t=0}^{\infty}$*, a stochastic equilibrium is defined as the time paths of consumption* $\{c_t, c_t^N, c_t^T\}_{t=0}^{\infty}$*, capital* $\{k_t^T, k_t^N\}_{t=0}^{\infty}$*, assets* $\{a_t\}_{t=0}^{\infty}$*, interest rate* $\{r_t\}_{t=0}^{\infty}$*, prices* $\{p_t, p_t^N\}_{t=0}^{\infty}$*, real wages* $\{w_t^N, w_t^T\}_{t=0}^{\infty}$ *and employment* $\{n_t^N, n_t^T\}_{t=0}^{\infty}$ *that satisfy in every period* t *the following*

- *Consumption Euler equation* (A.1)*;*
- *Optimal non-tradables and tradables consumption* (7)*;*
- *Price index equation* (8)*;*
- *Optimal level of capital in each sector* (A.4)*;*
- *Sectoral labor supply equations* (A.2)*;*
- *Sectoral labor demand equations* (A.3)*;*
- *Resource constraint for non-tradables* (18)*;*
- *Current account equation* (19)*;*
- *Interest rate equation* (20)*.*

Appendix B Model with capital and search frictions

In this section, I extend the baseline model by adding capital as an input to the production function.[11] I will present only aspects which differ from the baseline model.

[11] As discussed by Shimer (2010) and Blanchard and Galí (2010), introducing capital matters for the reaction to productivity shocks especially when income and substitution effects cancel out.

In addition to labor employed, the representative firm operating in sector $j \in \{N, T\}$ owns capital k_t^j which depreciates at rate δ. The firm combines the labor of production workers with capital to produce good j sold at price p^j using the following Cobb-Douglas production function:

$$y_t^j = z_t^j (k_t^j)^\alpha (n_t^j (1 - v_t^j))^{1-\alpha}$$

where z is total factor productivity. The general good is used both for consumption and investment. The representative firm's optimization problem in each sector is similar to the baseline model presented in section 4 with an additional state variable k_t^j and the choice of current period investment i_t^j such that

$$J^j(\mathcal{S}_t^j) = \max_{v_t^j \in [0,1], i_t^j} p_t^j y_t^j - w_t^j n_t^j - p_t^j i_t^j + \tilde{\beta} \mathbf{E}[J^j(\mathcal{S}_{t+1}^j)]$$

subject to:

$$\begin{aligned} y_t^j &= z_t^j (k_t^j)^\alpha (n_t^j (1 - v_t^j))^{1-\alpha} \\ k_{t+1}^j &= (1 - \delta) k_t^j + i_t^j \\ n_{t+1}^j &= n_t^j (v_t^j q_t^j + 1 - \rho_j) \\ \log z_{t+1} &= (1 - \rho_z) \log \bar{z} + \rho_z \log z_t + \sigma_z \varepsilon_{t+1} \end{aligned}$$

where $\mathcal{S}_t^j = \{z_t^j, n_t^j, k_t^j\}$.

The optimal level of capital is

$$p_t^j = \tilde{\beta} \mathbf{E} \left[p_{t+1}^j \alpha z_{t+1} \left(\frac{k_{t+1}^j}{n_{t+1}^j (1 - v_{t+1}^j)} \right)^{\alpha - 1} + p_{t+1}^j (1 - \delta) \right] \tag{B.1}$$

where capital in sector j is produced using the same production technology as good j.

Similar to the baseline model, for each sector j I get the equation for the marginal value of labor

$$J_n^j(\mathcal{S}_t^j) = p_t^j (1 - \alpha) z_t^j \left(\frac{k_t^j}{n_t^j (1 - v_t^j)} \right)^\alpha \left(1 + \frac{1 - \rho^j}{q_t^j} \right) - w_t^j$$

as well as the inter-temporal equation determining the optimal choice of the share of recruiters

$$p_t^j(1-\alpha)z_t\left(\frac{k_t^j}{n_t^j(1-v_t^j)}\right)^{\alpha} = q_t^j\tilde{\beta}\mathbf{E}\left[p_{t+1}^j(1-\alpha)z_{t+1}^j\left(\frac{k_{t+1}^j}{n_{t+1}^j(1-v_{t+1}^j)}\right)^{\alpha}\left(1+\frac{1-\rho^j}{q_{t+1}^j}\right)-w_{t+1}^j\right] \quad \text{(B.2)}$$

As in the baseline version, sectoral wages are determined at the beginning of each period through Nash bargaining between firms and households which results in the wage equation

$$w_t^j = \phi p_t^j(1-\alpha^j)z_t\left(\frac{k_t^j}{n_t^j(1-v_t^j)}\right)^{\alpha^j}(1+\theta^j)-(1-\phi)p_t\frac{U_n(c_t,n_t)}{U_c(c_t,n_t)} \quad \text{(B.3)}$$

Introducing capital changes the resource constraint for the non-tradable goods sector to

$$y_t^N + (1-\delta)k_t^N = c_t^N + k_{t+1}^N \quad \text{(B.4)}$$

and the trade balance to

$$tb_t = y_t^T + (1-\delta)k_t^T - k_{t+1}^T - c_t^T + p_t^C y_t^C,$$

while the current account is given by

$$ca_t = a_{t+1} - a_t = r_t a_t + tb_t. \quad \text{(B.5)}$$

Definition 4 *Given the exogenous path of sectoral productivities* $\{z_t^N, z_t^T\}_{t=0}^{\infty}$ *and commodity price* $\{p_t^C\}_{t=0}^{\infty}$*, a stochastic equilibrium is defined as the time paths of consumption* $\{c_t, c_t^N, c_t^T\}_{t=0}^{\infty}$*, capital* $\{k_t^T, k_t^N\}_{t=0}^{\infty}$*, assets* $\{a_t\}_{t=0}^{\infty}$*, interest rate* $\{r_t\}_{t=0}^{\infty}$*, prices* $\{p_t, p_t^N\}_{t=0}^{\infty}$*, real wages* $\{w_t^N, w_t^T\}_{t=0}^{\infty}$ *and labor market measures* $\{n_t^N, n_t^T, v_t^N, v_t^T, u_t^N, u_t^T, \theta_t^N, \theta_t^T\}_{t=0}^{\infty}$ *that satisfy in every period the following*

- *Consumption Euler equation* (9)*;*
- *Optimal non-tradables and tradables consumption* (7)*;*
- *Price index equation* (8)*;*
- *Optimal sectoral capital* (B.1)*;*
- *Sectoral labor market tightness* (1)*;*
- *Laws of motion of sectoral employment* (2)*;*

- *Sectoral allocation of job search* (10)*;*
- *Sectoral job creation equations* (B.2)*;*
- *Sectoral wage equations* (B.3)*;*
- *Aggregate labor market clearing condition* (3)*;*
- *Resource constraint for non-tradables* (B.4)*;*
- *Current account equation* (B.5)*;*
- *Interest rate equation* (20)*.*